LEEDS PHILOSOPHICAL AND LITERARY SOCIETY.

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A. A. COMMON, LL.D., F.R.S., Vice President, in the Chair.

Professor Henry C. Lord, Director of the Emerson McMillin Observatory, Ohio State University, Columbus, Ohio, U.S.A.,

was balloted for and duly elected a Fellow of the Society.

The following Candidate was proposed for election as a Fellow of the Society:—

William James Stewart Lockyer, M.A., Ph.D., 16 Penywern Road, South Kensington, S.W. (proposed by J. W. L. Glaisher).

Eighty-one presents were announced as having been received since the last meeting, including, amongst others:—

Cayley's Collected Mathematical Papers, vol. xii., presented by the Cambridge University Press; Cambridge Observatory, Astronomische Gesellschaft Catalogue, Zone $+25°$ to $+30°$, presented by Sir R. S. Ball; S. Glasenapp, Mesures Micrométriques d'Etoiles Doubles, presented by the St. Petersburg Astronomical Observatory; Harvard Observatory Annals, vol. xxviii., part 1, and vol. xxxvi., presented by the Observatory; L. O. Hesse's Gesammelte Werke, presented by the Munich Academy; G. Müller, Photometrie der Gestirne, presented by the author; E. C. Pickering, Navigation in fog, presented by the author; E. J. Stone and H. H. Turner, Tables for facilitating computa-

tion of star constants, presented by Professor Turner; C. A. Young, The Sun, new edition, presented by the author; Photographs of Spectra taken with the Bruce spectroscope, presented by Mr. Newall; Various volumes of MS., presented by Messrs. W. Wesley and Son.

Note on the Mean Motions of the Lunar Perigee and Node.
By Professor Ernest W. Brown, Sc.D.

The values given in my paper in the *Monthly Notices* 1897 March, for the mean motions of the perigee and node require slight corrections to make the comparison with Hansen's values complete. Although these corrections are much less than the possible errors which I thought it necessary to assign, and may be supposed to be included therein, there is a perceptible effect on the differences between the various results in the case of the perigee, and further my theoretical result for it comes much closer to that of Hansen. The nature of the correction is as follows:—

The disturbing function is of the form $m'Q$, where m' is the mass of the Sun and Q is a function of the solar and lunar coordinates. As is usual, we put $m'=n'^2a'^3$, instead of $m'=n'^2a'^3-E-M$, where n', a' are the solar mean motion and mean distance, and E, M, the masses of the Earth and Moon. Hansen uses the latter, which is the correct substitution for m'.

The correction is in general sufficiently accounted for if we multiply all the terms of the first order of the disturbing forces by

$$1-\frac{E+M}{m'}=1-\frac{1}{330000}, \text{ approximately;}$$

those of the second order by

$$\left(1-\frac{E+M}{m'}\right)^2=1-2\frac{E+M}{m'}, \text{ approximately.}$$

It is in all cases sufficient to consider the terms of the first order only in making this correction, except in the motion of the perigee, where the terms depending on the square of the disturbing forces are about equal in magnitude to those depending on the first power.

Hence, putting ϖ_1, θ_1 for the mean motions of the perigee and node, the corrections will be, with a sufficient approximation,

$$-\frac{1}{2}\varpi_1\cdot\frac{E+M}{m'}-\frac{1}{2}\varpi_1\cdot 2\frac{E+M}{m'}=-\frac{3}{2}\cdot\frac{1}{330000}\varpi_1,$$

and

$$-\theta_1\cdot\frac{E+M}{m'}=-\frac{1}{330000}\varpi_1,$$

respectively.

The theoretical values of the annual mean motions, given in the paper, and there numbered (i), are

$$+146\ 435''{\cdot}3 \pm 1''{\cdot}8, \qquad -69\ 679''{\cdot}5 \pm 1''{\cdot}1.$$

The corrections examined above are therefore

$$-0''{\cdot}7, \qquad +0''{\cdot}2;$$

making my theoretical values

$$+146\ 434''{\cdot}6 \pm 1''{\cdot}8, \qquad -69\ 679''{\cdot}3 \pm 1''{\cdot}1.$$

The former is now quite close to the corrected value of Hansen—that previously numbered (iv)—which is 146 434″·9. The difference from the observational value is now 1″·0, but, as I remarked before, no argument can be based on this. The argument in the case of the node is quite unaffected; the error of 2″·0 in Hansen's theoretical result remains.

Haverford College, Pa.:
1897 *May* 1.

On some Spectroscopic Determinations of Velocity in the Line of Sight made at the Cambridge Observatory. By H. F. Newall.

The present note gives some results recently obtained in determinations of velocity in the line of sight with the Bruce spectroscope attached to the 25-inch refractor (the Newall telescope) of the Cambridge Observatory (*Monthly Notices*, lvi. p. 98).

The instrument was used for determinations of velocity and for the careful study of a few spectra throughout the year 1896; but on account of more or less systematic differences between the results obtained from the photographs and the results obtained by Dr. Vogel at Potsdam, it was necessary to subject the instrument to an even more searching investigation than it received in the course of the first adjustment. The difficulty has been the setting of the comparison spectrum, and the investigation has been of a very troublesome kind, as has already been indicated in the annual reports (see present vol. *Monthly Notices*, p. 250). The instrument is now apparently in excellent adjustment. The test photographs are completely satisfactory, and moreover the velocities deduced for several bright stars are in good agreement with those deduced by Dr. Vogel from photographs taken with the Potsdam spectrograph.

The cause of the systematic discrepancies above referred to has been discovered. A minute deflection of the collimator had been produced every time the comparison prisms were used. The deflection was traced to a deformation of a plate at the end of the strong frame of the spectroscope, brought about by altera-

tions in the pressure of a spring attached to it. It may be stated that the observed discrepancies in the determination of velocity—amounting to about 20 kilometres per second—would be accounted for by a movement of the slit of the spectroscope through about 7 thousandths of a millimetre. The deflection produced a shift of the lines in the comparison spectra relatively to the corresponding lines in the star spectrum. The unexpected position of the strain made it very difficult to trace, and the search was made all the more difficult by the fact that the effect was variable in magnitude.

The velocities quoted in the following table as examples of results recently obtained must not be regarded as more than provisional except in the case of *α Boötis*. More photographs are needed

α Tauri.

Plate C. 411. 1897 March 4, $9^h\ 48^m$. Exposure 30 minutes.

Temperature 4°·7 to 4°·1 C.

Wave-length of line.	Velocity deduced relative to Earth.	
λ4308	+87·2	
4326	+88·4	
4384	+78·3	
4405	+64·2	
4415	+75·7	
Mean ...		+78·8
Correction for Earth's orbital velocity ...		−29·6
Velocity relative to Sun		+49·2 kilometres per second.

[Vogel +47·6; Scheiner +49·4.]

α Orionis.

Plate C. 408. 1897 March 3. Exposure 55 minutes.

Wave-length of line.	Velocity deduced relative to Earth.	
λ4315	+29·3	
4370	+49·5	
4405	+37·3	
4415	+37·4	
Mean ...		+38·4
Correction for Earth's orbital velocity ...		−27·8
Velocity relative to Sun		+10·6 kilometres per second.

[Vogel +15·6; Scheiner +18·8.]

α Canis Minoris

(2 plates, 5 lines on each).

(1) Plate C. 405. 1897 Feb. 17, $12^h\ 10^m$. Exposure 36 minutes.

Wave-length of line.	Velocity deduced relative to Earth.
λ4261	− 3·6
4308	+13·3
4384	+23·3
4405	+14·8
4415	+16·3
Mean ...	+12·8
Correction for Earth's orbital velocity ...	−16·3

Velocity relative to Sun − 3·5 kilometres per second.

(2) Plate C. 409. 1897 March 3, $10^h\ 54^m$. Exposure 33 minutes.

Wave-length of line.	Velocity deduced relative to Earth.
λ4261	+ 7·9
4308	+17·0
4384	+14·8
4405	+17·9
4415	+25·2
Mean ...	+16·6
Correction for Earth's orbital motion ...	−21·5

Velocity relative to Sun − 4·9 kilometres per second.

[Vogel −7·9; Scheiner −10·5.]

β Geminorum.

(1 plate, 5 lines).

Plate C. 414. 1897 March 19, $9^h\ 53^m$. Exposure 45 minutes.

Wave-length of line.	Velocity deduced relative to Earth.
λ4261	+22·8
4308	+28·0
4384	+23·5
4405	+28·9
4415	+29·3
Mean ...	+26·5
Correction for Earth's orbital motion ...	−27·2

Velocity relative to Sun − 0·7 kilometre per second.

[Vogel +1·9; Scheiner +0·4.]

γ Leonis, bright component

(2 plates, 3 lines each).

(1) Plate C. 419. 1897 March 24, $10^h\ 47^m$. Exposure 90 minutes.

Wave-length of line.	Velocity deduced relative to Earth.
λ4261	−21·7
4405	−25·4
4415	−28·8
Mean ...	−25·3
Correction for Earth's orbital motion ...	−17·0
Velocity relative to Sun	−42·3 kilometres per second.

(2) Plate C. 423. 1897 April 10, $11^h\ 13^m$. Exposure 61 minutes.

Wave-length of line.	Velocity deduced relative to Earth.
λ4370	−13·0
4405	−20·2
4415	−10·0
Mean ...	−14·4
Correction for Earth's orbital velocity ...	−23·1
Velocity relative to Sun	−37·5 kilometres per second.

[Vogel −36·5; Scheiner −40·5.]

Test Photographs.

Diffused Sunlight from Cloud.

(1) Plate C. 407*d*. 1897 March 2.

Wave-length of line.	Velocity deduced relative to Earth.
λ4261	−7·6
4272	+1·7
4308	+1·3
4384	−6·7
4405	+0·2
4415	−0·7
Mean ...	−1·9 kilometre per second.

(2) Plate C. 419*a*. 1897 March 27.

Wave-length of line.	Velocity deduced relative to Earth.
λ4261	−1·2
4308	+5·8
4326	−0·6
4384	−0·6
4405	+1·8
Mean	... +1·0 kilometre per second.

The velocity of *α Boötis* has been determined also by Belopolsky and Keeler, and accordingly this star was selected for special study, and the results are given in the table on page 573.

In taking the photographs, the following procedure is adopted. The slit is set parallel to the direction of daily motion and is partly covered by suitable diaphragms, which are movable, so that parts of the slit may be covered or exposed at the will of the observer. An electric spark between iron points is used as the terrestrial source of light for the comparison spectrum. First two exposures are made for the spectrum of iron ; next the star spectrum is taken ; and after the exposure for the star, two more exposures are made for the iron. Thus five spectra are set side by side on the one plate.

Iron taken before the star
Iron taken after the star
Star spectrum
Iron taken before the star
Iron taken after the star

The outer iron spectra are used for adjusting the wire of the micrometer to parallelism with the lines in the spectrum, the wire being set to bisect a line in the pair of spectra taken either before or after the star spectrum. The measurements are made only on the two inner iron spectra and on the star spectrum. The width of the five spectra is so small, that the curvature of the lines is not appreciable—an advantage largely due to the fact that only one prism is used. The width of the star spectrum is about 0·4 millimetre, and this is attained when the star-trail backwards and forwards along the slit amounts to about 0·2 millimetre.

The light from the spark passes into the slit after being reflected three times by reflecting prisms, and after passing through a lens. That satisfactory results are obtained from the

use of a spark situated off the *real* axis of the collimator is attributed in part to the convenience of the reflecting slit (Dr. Huggins's arrangement), by which the observer is enabled to see the image of the whole spark whilst it is being adjusted on the slit, and in part to careful adjustment of the reflecting prisms and to exceedingly careful focussing of the collimator.

The following method of focussing the collimator has been found by far the most satisfactory. The collimator is focussed first by visual observations, carried out according to Schuster's method, the whole aperture being used. Then a diaphragm is put on, consisting of an opaque card with a circular aperture half that of the collimator lens. The collimator of the Bruce spectroscope has an aperture of two inches. Hence the diaphragm lets a one-inch beam pass. A photographic plate is put into the camera of the spectroscope, and two exposures are made on the same plate, one being got when the one-inch beam passes through the thick part of the prism, the other when the one-inch beam passes through the thin part. If the collimator is properly focussed, there is no shift or displacement of the lines in the two spectra. The existence of shift is a sign that the collimator is not adjusted (assuming that the prism and lenses are not faulty). The rationale of the method is clear, and when the collimator has been focussed in this manner the definition of the photographs is very good over a very considerable range of spectrum ; the aberrations due to unsymmetrical passage through the prism when the incident beam is not properly collimated being very noticeable. Moreover, when the collimator has been focussed in the way described, a slight maladjustment of the axis either of the pencil of starlight or of the pencil of iron-spark-light relative to the axis of the collimator appears to be of no importance.

The results obtained in the case of *α Boötis* are set forth in the following table. Entirely independent measurements have been made by myself (N.) and by my assistant, A. W. Goatcher (G.), from seven plates of *α Boötis* taken since the last alterations were made to the spectroscope.

The scale of the photographed spectra is such that one millimetre in the length of the spectrum corresponds to 20·8 tenth-metres in the neighbourhood of λ 4260 and to 24·2 tenth-metres in the neighbourhood of λ 4415. The microscope used in measuring the photographs has a magnifying power of 21, and the micrometer screw is an excellent screw by Cooke. The average value of a revolution of the micrometer head expressed in terms of kilometres per second is 385 km/sec.

The velocity of light has been taken as 3×10^5 km/sec, and the solar parallax adopted is 8″·838.

α Boötis.

(7 plates).

Measurements made by N.

Plate. Date 1897.	C. 422. Apr. 8^d13^h.	C. 424. Apr. 10^d13^h.	C. 427. Apr. 14^d13^h.	C. 428. Apr. $14^d13\frac{1}{2}^h$.	C. 429. Apr. 27^d12^h.	C. 430. May 1^d12^h.	C. 433. May 3^d13^h.
λ4250·3	...	− 9·7	...	...	...	...	...
4250·9	...	− 3·9	...	...	...	...	...
4260·6	−21·2	−11·0	− 8·3	−16·1	+ 1·0	0·0	− 1·6
4308	− 8·0	− 2·1	+ 2·1	− 9·5	−4·0	−1·1	+ 7·8
4315	...	...	−11·9	...	...	...	...
4370	− 5·9	− 1·9	− 1·0	−12·3	...	−3·2	...
4384	−12·9	− 8·0	− 9·2	− 7·7	−6·6	...	...
4405	− 3·3	− 6·7	− 3·9	+ 0·8	−3·5	+8·2	+13·1
4415	− 6·7	− 3·3	+ 0·5	− 5·8	−3·0	+6·7	+13·8
4427	−15·4	− 2·8	+ 0·3	− 2·2	...	...	+10·4
Mean ...	−10·5	− 5·5	− 3·9	− 7·5	−3·2	+2·1	+ 8·7
Corr. to ⊙	+ 1·9	+ 1·0	− 0·7	− 0·7	−6·2	−7·9	− 8·7
Vel. rel. to ⊙	− 8·6	− 4·5	− 4·6	− 8·2	−9·4	−5·7	0·0
Prob. error	± 1·5	± 0·8	± 1·2	± 1·5	±0·8	±1·5	± 2·4

Mean for 7 plates − 5·9 ± 0·8 (N)

Measurements made by G.

λ4250·3	−13·8	−14·7	...	...	...	...	...
4250·9	...	− 3·1	...	...	...	...	...
4260·6	−17·4	−14·2	−13·3	−16·6	−3·7	0·0	− 3·5
4308	− 7·5	− 2·3	− 0·3	− 6·4	...	+2·8	+ 8·4
4315	...	...	−12·0	...	−4·0	...	...
4370	− 5·2	− 4·6	− 2·2	−14·2	...	...	...
4384	−20·6	− 9·0	− 6·9	− 7·7	−4·7	...	...
4405	−12·0	− 4·1	− 6·2	− 3·0	+0·2	+2·7	+ 7·1
4415	− 8·1	− 1·0	− 0·2	− 2·0	+0·5	+2·5	+11·5
4427	− 8·9	−16·1	− 4·0	− 0·3	...	...	...
Mean ...	−11·7	− 7·7	− 5·6	− 7·2	−2·3	+2·0	+ 5·9
Corr. to ⊙	+ 1·9	+ 1·0	− 0·7	− 0·7	−6·2	−7·9	− 8·7
Vel. rel. to ⊙	− 9·8	− 6·6	− 6·3	− 7·8	−8·6	−5·9	− 2·8
Prob. error	± 1·3	± 1·3	± 1·2	± 1·6	±0·7	±0·5	± 2·2

Mean for 7 plates −6·8 ±0·6 (G)

[Vogel −7·0; Scheiner −8·3; Belopolsky −5·7; Keeler −6·8.]

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The lines measured have been selected in the star spectrum from a great number of iron lines, on account of ease in identification. A comparison of the solar spectrum, as photographed with small dispersion, and as represented in Rowland's map, which has a dispersion about sixty-six times as great, shows that the iron lines are involved in groups, so that the choice of lines is practically decided by the predominance of an iron line amongst its immediate neighbours. The *group* is bisected in the photograph, but the bisection does not necessarily coincide with the true position of the iron line in the group. Consequently each line chosen will give a velocity which is affected with an error in identification. If enough lines are measured, these errors might be supposed to cut one another out. The discussion of the measurements of *α Boötis* will show what is to be expected. (See the table below.)

The determination of velocity from a single line is based on eleven micrometer settings—three on the iron line in the spectrum taken at the beginning of the exposure, and three on the iron line taken at the end, and five settings on the group which represents the iron line in the star spectrum. The difference between the means for the iron lines is the shift due to change of temperature or to change in the position of the instrument during the exposure. It is assumed that this change is a linear function of the time, and the displacement of the star line or group relative to the mean of the initial and final positions of the iron line is taken to get the velocity. The mean of about seven velocities deduced from as many lines in a plate is regarded as the velocity deduced from the plate.

It appears from the measurements that the probable error of the velocity determined from a single line (eleven micrometer settings), on the assumption of no error of identification, is between $\pm$ 0·2 and $\pm$ 0·6 km/sec according to the goodness of the photograph.

The probable error of the velocity deduced from about seven lines on a plate is between $\pm$ 0·5 and $\pm$ 1·5 km/sec, again assuming no errors in identification.

The probable error of the velocity deduced from seven plates is about $\pm$ 0·7, on the assumption that *α Boötis* has a constant velocity relative to the Sun, and is not a spectroscopic double-star.

With a view of seeing how far the velocity deduced from a given line on the seven plates would give evidence of a calculable error of identification, the mean velocity relative to the Sun for each of seven lines was got out.

The group of lines, which was bisected in the micrometer setting, was compared in each case with Rowland's map ; and the attempt was made to form an estimate of the displacement of the centre of the group from the true centre of the iron line which predominated in the group, weight being given to the foreign lines involved in the group according to the intensity of

the lines as recorded in Rowland's tables. This displacement was expected to be in a sense the error in identification of the true iron line in the star spectrum. The velocity corresponding to the displacement was then computed, and the results are entered in the last column of the table. The assumption underlying this, viz., that the spectrum of *α Boötis* is similar in detail to that of the Sun, appears to be supported by a comparison of the spectra.

	Velocity relative to Sun. N.	G.	Residuals. N.	G.	Computed Error of Identification.
λ4260·6	−11·0	−12·7	+5·4	+6·0	+7·7
4308	− 5·2	− 3·3	−0·5	−3·4	+5·6
4370	− 5·6	− 6·0	−0·1	−0·7	0·0
4384	− 9·0	−10·6	+3·3	+3·9	0·0
4405	− 2·7	− 5·1	−3·0	−1·6	+1·4
4415	− 2·8	− 2·5	−2·9	−4·2	−4·1
4427	− 3·7	− 6·8	−2·0	+0·1	+4·1
Mean ...	− 5·7	− 6·7 km/sec	Mean		... +2·5 km/sec

The material is in reality too slight for a completely satisfactory comparison; and yet as far as it goes, it affords means of estimating the errors involved in the use of small dispersion. One point is made clear by the investigation, viz. that little or no advantage, so far as the elimination of the error of identification is concerned, would be gained by *doubling* the dispersion and resolving power of the spectroscope; for the disturbing lines in the groups selected would not be separated from the iron lines without a considerably greater increase.

α Geminorum (*Castor*).

Two excellent photographs of the spectra of the components of α Geminorum were obtained in 1896 November, each having a spectrum of the faint star in the system between two spectra of the bright star. It was observed that (i.) the spectrum of the bright star was similar to that of *α Lyræ*; (ii.) the spectrum of the faint star resembled that of *α Cygni* very closely; (iii.) lines common to the two spectra showed considerable relative shift; (iv.) the shift of the lines in the faint star spectrum was towards the red in one photograph and towards the blue in the other.

The photographs have been measured with the following results:—

(1) Plate C. 382. 1896 Nov. 4, $13^h\ 35^m$ to $15^h\ 9^m$. Observer, N.

Exposures	Bright component	30 minutes.
	Faint „	50 „
	Bright „	12 „

	Velocity of Faint relative to Bright.
λ4290	−50·3
4326	−35·2
4395	−41·1
4482	−41·8
4550	−43·0
Mean ...	−42·3 km/sec. (N)

(2) Plate C. 385. 1896 Nov. 5, $11^h\ 47^m$ to $13^h\ 14^m$. Observer, G.

Exposures { Bright component ... ... 25 minutes. Faint ,, ... 50 ,, Bright ,, ... 10 ,,

	Velocity of Faint relative to Bright.
λ4290	+28·9
4308	+29·6
4352	+37·1
4418	+32·3
4482	+23·0
4550	+33·8
Mean ...	+30·8 km/sec. (N)

These photographs were taken without any comparison spectra last autumn, before the cause of difficulty with the comparison spectra was found out; but it is clear that they are not open to the objection that has necessitated the rejection of results relating to the determination of velocity with the Bruce spectroscope before 1897 February. But as the cause of vitiation was not known, the large shifts observed in the spectra of *Castor* now dealt with were regarded with a suspicion that is now removed. The velocities deduced are, therefore, regarded as completely trustworthy.

Belopolsky has found (*Astrophysical Journal*, vol. v. p. 1, 1897) that the faint component of *Castor* is a spectroscopic binary, and that within a period of 2·91 days the velocity of the star relative to the Sun passes through a series of changes, ranging between +35 and −45 km/sec. Belopolsky has deduced as the time of periastron passage the date 1896 February $28{\cdot}82 \pm 2{\cdot}91\ n$. Assuming this period, the interval since the preceding periastron passage for each of the photographs above referred to is given below; the corresponding velocities deduced from Belopolsky's orbit are also given. Assuming that the velocity (relative to the Sun) of the system a_1 (faint star and invisible companion) is the same as that of a_2 (bright

star), then the observed velocities above recorded should be comparable with those computed from Belopolsky's data.

			$\frac{dz}{dt}$ km/sec.	Cambridge Obs.
C. 382	1896 Nov. 4·60	$= 1896$ Feb. $28^{d}\cdot82 + (2^{d}\cdot91 \times 85) + 2^{d}\cdot44$	+39	−42·3
C. 385	1896 Nov. 5·53	$= 1896$ Feb. $28^{d}\cdot82 + (2^{d}\cdot91 \times 86) + 0^{d}\cdot46$	−39	+30·8

The photographs show a change of velocity quite comparable with that discovered by Belopolsky, but exactly opposite in phase. The period $2^{d}\cdot91$ is not accurately enough known for the calculation of an ephemeris so far ahead as seven months. It would appear that an odd number of half periods has been lost or gained between April and November. Belopolsky's discussion of thirty-two photographs pointed to some possible variation in the period, and it is to be hoped that he has secured more evidence in the present year.

The photographs obtained at Cambridge, however, appear to support Belopolsky's most interesting discovery, that in the system of *Castor* two periods must now be recognised, one about 1000 years, the other probably less than three days.

On a New Binary Star with a Period of $5\frac{1}{2}$ *Years* (β 883).

By T. J. J. See, A.M., Ph.D. (Berlin).

For several months I have been practically certain that the double star β 883=Lalande 9091, $\alpha = 4^{h}\ 44^{m}\ 33^{s}\cdot0$, $\delta = +10^{\circ}\ 52'$ (1875·0) is revolving in a period of about 5·5 years, and thus describing its orbit in less than one-half the time required for the most rapid system heretofore known; but it seemed desirable to withhold any announcement of such an extraordinary result until the observations should remove every reasonable doubt of the correctness of the conclusion. Indeed, this hypothesis first presented itself to my mind in the summer of 1895, when I was examining the orbits of all known binaries with a view to selecting those most accurately determined for incorporation into the work recently published (*Researches on the Evolution of the Stellar Systems*, vol. i.).

Professor Schiaparelli had very kindly communicated by private letter a number of his unpublished obvervations of this star, and on combining these with other measures in my possession it seemed clear that only a period of about 5·5 years would satisfy the observations. Professor Burnham, however, did not regard the evidence as conclusive, and I decided to wait till further measures were available. On entering upon the measurement of southern double stars I felt that notwithstanding the northern position of this object the great interest attach-

ing to it would justify occasional examination with the Lowell 24-inch refractor. Accordingly I observed it at Flagstaff in September, and have since continued the observations every month, except when the Observatory was in process of removal to Mexico. These observations since 1896 September furnish the key to the situation ; and as the Sun is now rapidly overtaking the star, and it cannot be followed much longer this year, the present is a favourable time to present to the Society the results at which we have arrived. In the sweeps for new double stars I am assisted by Mr. W. A. Cogshall, S.B., who generally examines all objects of difficulty, and at different times has secured a considerable number of measures. His work has been done with care and precision, and I have valued his judgment in all cases of doubt. In our observations of this star we are in entire accord.

Individual Observations of β 883.

t	θ_0	ρ_0	Observers.	Remarks.
	°	"		
1879·00	[197·5] 17·5	0·35	Burnham	
1887·096	84·3	0·15	Schiaparelli	Very difficult.
·170	85·0	0·15	,,	,, ,,
·172	84·7	0·15	,,	,, ,,
1888·055	119·5	0·18	,,	
·066	129·4	0·18	,,	
·142	...	...	,,	Simple.
1889·017	137·0	...	,,	Doubtful.
·065	...	...	,,	Simple.
·176	158·8	0·15	,,	
·192	158 3	0·20	,,	
·195	144·2	0·25	,,	+180°·0 ?
1890·130	200·7	0·15	,,	Doubtful.
·132	206·9	0·15	,,	,,
·149	202·8	0·18	,,	,,
1891·14	Single	Many nights	Burnham	
·146	...	...	Schiaparelli	Single.
·149	...	...	,,	,,
·785	125·2	0·12	Burnham	
1892·057	116·4	0·06	,,	
·060	127·8	0·12	,,	
1893·072	...	...	Schiaparelli	Single.
1894·146	...	...	,,	,,
·165	...	...	,,	,,
·65	173·5	0·20	Barnard	

t	θ_o	ρ_o	Observers.	Remarks.
	°	″		
1895·146	...	...	Schiaparelli	Uncertain.
·745	186·0	0·43	See	Good images.
·746	186·8	0·35 ±	,,	Separated.
·745	188·9	...	Moulton	,,
1896·692	9·9	0·17 ±	See	Distinctly elongated, but no trace of separation.
·693	12·0	0·14 ±	,,	
·728	28·7	0·25 ±	,,	Elongated, but not separated.
·843	22·8	0·18 ±	,,	Elongation very distinct.
·844	23·8	0·17 ±	,,	Notched under power of 1000.
1897·028	34·6	0·28 ±	,,	Well separated, but very close.
·118	34·2	0·28	,,	Distinctly separated. Good measures.
·162	35·5	0·25	,,	Nicely separated.
·162	36·0	0·35	Cogshall	
·192	37·8	0·35	See	Distinctly separated, with black line between.
·193	37·1	0·35 ±	,,	Distinctly separated, with black line between.
·192	35·5	...	,,	Angle certainly not smaller than this value.

In September last the star was moderately elongated in angle 11°, but no trace of separation could be seen, though examined under good seeing with a power of 1000; now it is distinctly separated with a black line between the components, with a power of only 500, and cannot be regarded as a very difficult object for the great Telescope.

The distance at present can hardly fall short of 0″·32, while in September it certainly was not more than half this value.

This object was discovered by Burnham with the Chicago 18½-inch Clark refractor in 1879·0, and measured on one night. The components of the close pair (there is a third component in 152°·2, 18″·22, 13th magnitude) have always been noted as nearly equal (generally as 7·8 and 8), differing at most by one or two tenths of a magnitude; and hence I suspect that the first angle given as 17°·5 ought to be increased by 180°. If the stars are sensibly equal the observers almost involuntarily adopt the smaller of the two possible position-angles. Our recent observations give an additional ground for this correction by showing that the stars are not separated in angle 17°·5, and hence it is improbable that the distance would have been given as 0″·35.

The observations of Schiaparelli in 1887, 1888, 1889, 1890, show that the motion is rapid, and indicate that the period is only a few years. In 1891 the star was single to Schiaparelli and also to Burnham, who examined it on several nights; which would imply that the companion was then in periastron.

In 1891·97 it was observed by Burnham, who concluded that 100° had been swept over by the radius vector since 1879·0. In 1894·65 Professor Barnard observed it, at my request, and found the angle to be 173°·5. While observing double stars at Madison in September 1895, I took occasion to measure this object, and found that it was close, but nicely separated with the 15½-inch refractor.

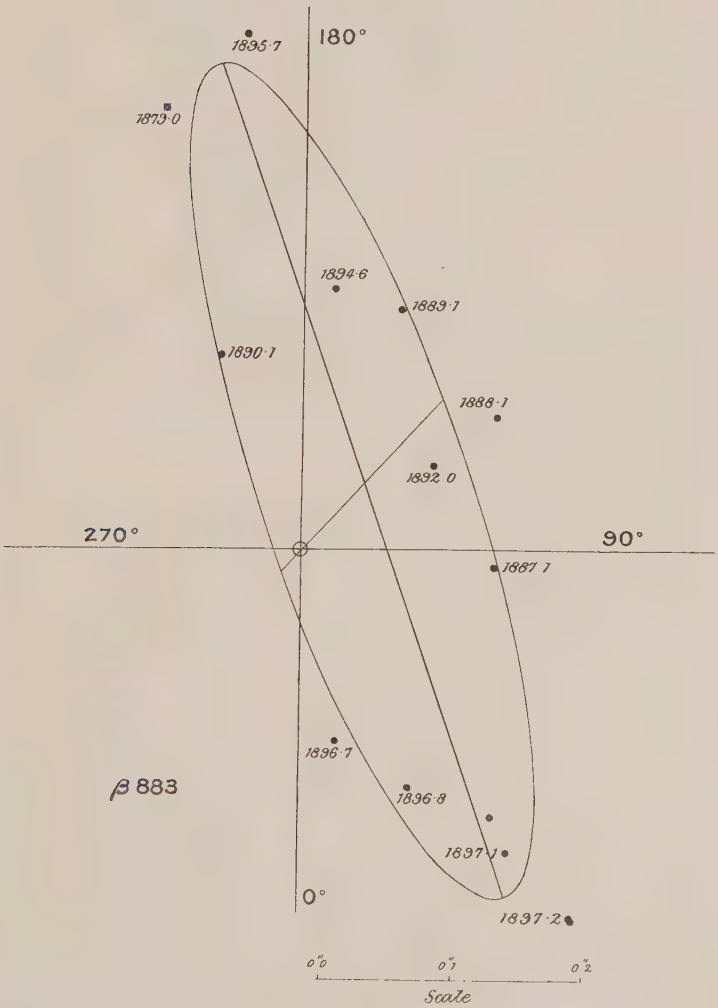
Mr. Moulton's measure was in substantial agreement with my own, and neither of us regarded it as a very difficult object for a good telescope. I have no record of any subsequent measures until the beginning of our work at Flagstaff. In September it certainly was slightly elongated, but not a trace of separation could be seen. In our subsequent observations we were equally unable to get evidence of separation until November 4, when with a power of 1000 and good seeing there appeared for the first time some evidence of "notching." As soon as observations were begun in Mexico I took occasion to examine this object. It then appeared for the first time actually separated, though still very close. In the subsequent observations the object became easier and easier, until now the star is well separated with a black line between the components, and the division can be seen with comparative ease even if the observer has little experience or the seeing is not the very best. We estimate the seeing on a scale of 10; that at Flagstaff was uniformly rated at 7; here in Mexico it has been rated at 4 and 5. The atmospheric conditions here were certainly not so good as those at Flagstaff.

Under these circumstances I do not hesitate in the belief that the companion has recently passed periastron, and that we have observed it widen out from month to month, until it is now within the reach of moderate sized telescopes. I have, therefore, investigated the general nature of the orbit, and herewith submit an approximate set of elements, which will be sufficiently exact for the use of observers.

The following considerations enable us to derive the essential element of the apparent orbit.

1. The motion since 1895·7, over an arc of 210°, indicates that the period will be very short, probably not far from five years; the observations of Schiaparelli point to a similar conclusion, and hence if we take the angle in 1879·0 to be 197°·5 we may conclude that three revolutions have been performed between that date and 1896; whence we deduce a period of about 5·5 years. All the observations are consistent with such a period except that of Burnham in 1891·97, where the angle appears to be in error about a quadrant. On inquiring of him whether there could be an error in the reduction of the angle, he writes that he does not think there can be an error of a quadrant in the published measure.

We shall at present make no attempt to assign the cause of the discrepancy. The other observations indicate an error of


180°
1895·7
1879·0
1894·6
1889·7
1890·1
1888·1
1892·0
270°
90°
1887·7
1896·7
β 883
1896·8
1897·1
0°
1897·2
0″·0
0″·1
0″·2
Scale

some kind, and we must reject either the one observation or all the others combined.

Even if all the other observations be discarded there will still be a case of rapid motion, as the angle is now 90°, or 270° different from that recorded in 1891·97.

2. The elements of the orbit are as follows :—

P = 5·5 years	$a = 0''·621$
T = 1896·40	$\Omega = 20°·6$
$e = 0·760$	$i = 82°·52$
$\lambda = 273°·83$	

Apparent orbit :—

Length of major axis	$= 0''·67$
Length of minor axis	$= 0''·16$
Angle of major axis	$= 19°·5$
Angle of periastron	$= 318°·0$
Distance of star from centre	$= 0''·07$

The apparent orbit shows the general nature of the motion, and indicates very decidedly that the period cannot much surpass 5·5 years, and that the eccentricity and inclination are very high.

Comparison of Computed with Observed Places. β 883.

t	θ_o	θ_c	$\theta_o - \theta_c$	ρ_o	ρ_c	$\rho_o - \rho_c$	N.
1879·00	197°·5	191°·0	+ 6°·5	0″·35	0″·37	−0″·02	1
1887·15	84·7	54·9	+29·8	0·15	0·21	−0·06	3
1888·06	124·4	130·2	− 5·8	0·18	0·15	+0·03	2
1889·14	158·5	179·2	−20·7	0·20	0·31	−0·11	4
1890·14	203·4	195·5	+ 7·9	0·16	0·30	−0·14	3
1891·14	single	22·4	...	...	0·24	...	1
1891·15	„	22·4	...	...	0·24	...	2
1891·97	33·1 ? 123·1	36·1	− 3·0?	0·12	0·30	−0·18	3
1893·07	single	80·1	...	...	0·16	...	1
1894·65	173·5	178·8	− 5·3	0·20	0·31	−0·11	1
1895·15	uncertain	190·9	...	...	0·37	...	1
1895·74	186·4	193·3	− 6·9	0·39	0·35	+0·04	2
1895·74	188·9	193·3	− 4·4	...	0·35	...	1
1896·692	11·0	23·3	−12·3	0·15 ±	0 25	−0·10	2
1896·805	25·1	25·3	− 0·2	0·20 ±	0·31	−0·11	2
1897·073	34·4	29·6	+ 4·8	0·28	0·31	−0·03	2
1897·162	35·5	31·0	+ 4 5	0·25	0·31	−0·06	3
1897·162	36·0	31·0	+ 5·0	0·35	0·31	+0·04	1
1897·192	37 5	31·4	+ 6·1	0·35	0·31	+0·04	2

3. It is now clear that Mr. Burnham's supposition, that the motion from 1879·00 to 1892 amounted to 100°, fell far short of the truth. At that time it must have amounted to about two and one half revolutions. For during the interval since 1892 the companion has swept over an arc of 270°; even if it were admissible to suppose that it is now in the third quadrant instead of the first (indicated by our observations), we should still have a motion of 90° since 1892, which contradicts the hypothesis of slow motion. Supposing that only 90° have been described since 1892, the slow motion hypothesis is contradicted by the observations of Schiaparelli in 1887, 1888, 1889, and by our inability to see the pair separated in 1896 with the 24-inch, whereas the object was easily divided in 1895·7 with a 15½-inch telescope. Also, if the motion is slow the minimum change which can have taken place since 1892 (90°) requires that the orbit should be much extended in the first quadrant, and hence the star would have been wide and easy at the time of Schiaparelli's observations; he actually found it close and difficult and, except in the first case, in the second and third quadrants. It is, therefore, clear that the motion of this system is very rapid, and that no other hypothesis will explain the phenomena.

The results here submitted give only a first approximation to the orbit, and doubtless some improvement of the elements will soon be possible. This remarkable object ought to be followed from month to month during the whole time it is visible. By securing a closely connected series of observations we shall soon be able to render the orbit very exact.

The discovery of an object revolving in a period of 5·5 years is an achievement of some philosophic significance in the history of double star astronomy. In the time of Sir John Herschel the most rapid of known binaries was ζ *Herculis*, with a period of 35 years. Twenty years ago the remarkable object 42 *Comæ Berenices* had reduced the shortest period to about 25 years, and in 1887 δ *Equulei* brought it down to 11·5 years. κ *Pegasi* (β 989) has since been shown to revolve in a similar period.

In β 883 we have for the first time a visible binary with a period fairly approaching those of the spectroscopic binaries recently discovered, and we seem assured that at last a link has been found connecting the two classes of objects. It is probable that other stars will disclose even shorter periods, for there is no reason why there should not be close doubles with periods of a single year or less. It will be an interesting object of future research to fill in the intervening steps between visible binaries with periods of a few years and the spectroscopic binaries revolving in a few days or months.

The more critical inquiry into the motion of close doubles will commend itself to the attention of double star observers with great telescopes, and on the other hand it may be hoped that the study of the relative motion in line of sight of the com-

ponents of binaries like β 883 will be taken up by some of our great observatories equipped with powerful spectroscopic appliances.

Lowell Observatory, City of Mexico:
1897 *March* 12.

Since the above paper was forwarded to the Secretary I have received from Professor Barnard full details of his observations in 1894. These measures are very important, and since they are not all included in the above list, they are here condensed.

t	θ_o	ρ_o	Remarks.
1894·649	173°·5	0″·20	Seeing 4 (scale of 5); the stars are north and south; power, 1500.
1894·668	176·8	0·20	Seeing, 4–5; power, 1500.
1894·671	178·7	0·16	Seeing, 5; power, 1500. Stars very close.

The instrument used was of course the 36-inch refractor.

These measures leave absolutely no doubt of the rapid motion between 1892 and 1894, and are therefore in accordance with the results given in the foregoing paper.

Lowell Observatory, City of Mexico:
1897 *April* 7.

Results of Double-Star Measures with the 8-inch Equatorial at Windsor, New South Wales, in 1896.
By John Tebbutt.

Ref. No.	Star.	Observed Mags.	Approx. Place of Star 1896·0. R.A.	Dec. S.	Fraction of Year.	Position Angle.	No. of Obs.	Distance.	No. of Obs.	Mag. Power.	Eyes.	Hour Angles.		Weight 1 to 5.
			h m	° ′		°		″				h m	h m	
1	*p* Eridani	...	1 36	56 43	·986	223·3	10	7·63	8	300	P	0 46 W	1 40 W	4
2	,,	...	,,	,,	·988	223·6	10	7·77	6	230	R	2 48 W	3 9 W	3
3	,,	6, 6	,,	,,	·999	223·7	10	7·44	8	300	R	2 41 W	3 11 W	3
4	Lacaille 2145	...	6 2	48 27	·986	33·6	10	1·60	5	300	P	2 37 E	2 1 E	4
5	,,	...	,,	,,	·988	33·0	10	2·11	10	230	P	4 11 E	2 53 E	2
6	,,	...	,,	,,	·988	32·1	10	2·05	7	230	P	4 11 E	2 53 E	3
7	,,	...	,,	,,	·999	32·4	9	...	...	300	P	1 6 E	0 43 E	3
8	V Puppis	...	6 36	48 8	·988	319·3	10	13·20	7	230	R	2 54 E	2 25 E	3
9	γ Piscis Vol.	$4\frac{1}{2}$, 6	7 10	70 20	·323	299·6	10	13·74	7	230	P	2 17 W	2 53 W	4
10	,,	4, 6	,,	,,	·326	299·2	10	13·68	10	230	P	0 26 W	1 1 W	4
11	γ Centauri	...	12 36	48 23	·323	357·8	10	1·87	10	300	P	1 21 E	0 51 E	4
12	,,	...	,,	,,	·326	358·6	10	...	...	300	P	2 43 E	2 0 E	4
13	,,	...	,,	,,	·326	358·6	10	1·62	10	535	P	2 43 E	2 0 E	4

Ref. No.	Star.	Observed Mags.	Approx. Place of Star 1896·0. R.A.	Dec. S.	Fraction of Year.	Position Angle.	No. of Obs.	Distance.	No. of Obs.	Mag. Power.	Eyes	Hour Angles.		Weight 1 to 5
			h m	° ′		°		″				h m	h m	
14	,,	...	,,	,,	·329	357·4	10	...	...	300	P	2 10 E	1 33 E	5
15	,,	...	,,	,,	·329	357·6	10	1·73	10	535				
16	β Muscæ	...	12 40	67 32	·326	336·2	10	1·17	5	535	R	1 43 E	1 8 E	4, 2
17	,,	...	,,	,,	·329	336·5	10	1·32	6	535	R	1 29 E	0 59 E	5, 4
18	α Centauri	1, 2	14 33	60 24	·320	208·5	10	21·61	7	300	P	2 40 E	2 10 E	4
19	,,	1, 2	,,	,,	·326	208·5	10	21·36	10	535	P	2 49 E	2 10 E	4
20	A Scorpii	$4\frac{1}{2}$, $6\frac{1}{2}$	15 47	25 1	·320	279·7	10	2·88	10	230	P	4 39 E	4 8 E	4
21	,,	$4\frac{1}{2}$, 7	,,	,,	·323	279·3	10	...	...	300	P	3 49 E	3 9 E	4
22	,,	$4\frac{1}{2}$, 7	,,	,,	·323	280·7	10	2·90	7	300	R			
23	,,	$4\frac{1}{2}$, 7	,,	,,	·326	279·0	10	2·70	10	300	R	3 0 E	2 24 E	5

Remarks.

In the column headed "Eyes" P denotes that the line joining the observer's eyes was parallel to that joining the components, and R that these lines were at right angles. The column headed "Hour angles" gives the hour angles between which the measures were made, and that headed "Weight" gives the value of each result on a scale of 1 to 5, 1 denoting the worst and 5 the best possible conditions.

The components of 2, 3, and 13 were noted to be equal. The south component of 4 and 7 was certainly the brighter. The primary of 8 was yellowish, and the companion blue.

Windsor, N.S. Wales: 1897 *April* 30.

Nebula H I, 43 *Virginis*. By Professor Kavasjee D. Naegamvala.

The nebula was photographed on three successive nights (April 16, 17, and 18, 1896) with exposures ranging from one hour to two hours in the principal focus of a 16½in. reflector of 122 inches focal length.

The nebula is No. 4,594 in the *New General Catalogue*, and is described as "remarkable, very bright, very large, extremely extended, position-angle 92°, very suddenly much brighter in the middle with a nucleus." It is engraved in the *Philosophical Transactions for* 1833, plate xiv. fig. 50, and also in *Mem.R.A.S.* vol. xxxvi. plate v. fig. 22.

Sir John Herschel, in his "Observations of Nebulæ and Clusters of Stars" (*P. T.* 1833), remarks that "there is a faint diffused oval light all about it," and he is "almost positive that there is a dark interval or stratum separating the nucleus and general mass of the nebula from the light above." Mr. Lassell (*Mem.R.A.S.* xxxvi) with his much larger instrument describes it as a "long spindle-shaped nebula, with a dark band through its entire length." He, however, does not seem to have noticed the nucleus.

Of the two drawings, that of Sir J. Herschel much more resembles the photographs. The photograph taken 1896 April 18, distinctly showed that the nebula was *elliptical* with its plane highly inclined to the line of sight. (This plate was unfortunately destroyed during the final washing.) The nucleus is of a starry character, and much more pronounced than in Sir J. Herschel's drawing.

I hope to submit to the Society a photograph of the nebula at an early date.

May, 1897.

The Effect of the General Illumination of the Sky on the Brightness of Field at the Focus of a Telescope. By F. L. O. Wadsworth.

(*Communicated by the Secretaries.*)

The investigations of Lord Rayleigh have shown that the general illumination of the sky is probably due to light scattered by small (*i.e.* small in comparison with the wave-length of light) particles in our atmosphere, and that the intensity of the light so scattered varies inversely as the fourth power of the wave-length. It seems pretty certain that these particles are present in nearly, if not quite as great quantities, in the upper regions of the atmosphere as in the lower strata, and that they consequently cannot

be of an aqueous origin. The majority of them being at a considerable distance above the surface of the earth, the scattered light which ultimately reaches the eye after repeated changes of direction is of plane wave front, and the general illumination is consequently of the same character as would be produced by a distant uniformly luminous surface of infinite extent. The illumination at any point p, which is the same as that at any other point in the focal field of a telescope, due to such a surface, is, as is well known,

$$I_p = \int_{\infty}\int_{\infty} I\,d\xi\,d\eta = \int_{\infty}\int_{\infty} I\,dx'\,dy' \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where I is the illumination due to a single point s in the surface; $\xi\,\eta$, the coordinates of the point p, referred to the geometrical image of s, as origin; and conversely x', y', the coordinates of s, referred to the intersection of the optical axis through p, with the surface in which s is situated. The general principle of energy requires that the value of (1) should be equal simply to A, the area of the aperture of the telescope, when the apparent brightness of the surface is unity, or to k A when the apparent brightness is k. A general analytical proof has also been given by Stokes.* It follows, therefore, that the intensity of illumination of the focal field of a telescope, due to the light of the sky, is simply proportional to the area of the telescope aperture, and entirely independent of the focal length of the latter. In the theory of astronomical and astrophysical instruments, particularly those intended for photographic and spectrographic work, in which the greatest possible contrast between the field and the image of the body dealt with is desired, this is a point of great importance, which, strangely enough, has been hitherto entirely overlooked by all writers on the subject whose papers are accessible to me.† Some, indeed, have been led to very erroneous conclusions in regard to the form and size of instrument best adapted to particular purposes by a neglect of this relation between aperture and brightness of field. It has, therefore, seemed to me to be worth while to rediscuss somewhat at length the conditions of maximum efficiency in astro-photo-spectro-graphic work. The discussion as regards the purely astro-photographic side of the question is briefly outlined in the current (July ?) number of *Knowledge*, "On the Conditions which Determine the Limiting Time of Exposure of Photographic Plates in Astronomical Photography," and will be found more fully developed, from a mathematical side, in the next (August) number of the *Astrophysical Journal*, "On the Conditions of Maximum Efficiency in

* *Trans. Roy. Soc. Edin.* vol. xx. p. 317, 1853. See also "Wave Theory," *Enc. Brit.* vol. xxiv. § 11.

† The importance of this point in the case of photographic work is enhanced by the character of the sky illumination, which, owing to its varying as the fourth power of the wave-length, has a much greater effect on the photographic plate than on the eye.

Astro-Photographic Work," in which questions relating to astrophysical work, particularly stellar and nebular spectrography and coronal photography, both during and without an eclipse, are also taken up. Without entering at length into the argument here, the more important conclusions which have been reached may be summed up as follows:—

I. The absolute intensity of the image of a celestial object, and, therefore, the absolute photographic light action (product of intensity by time) for a given time of exposure, will vary, (1) for extended sources as the square of the angular aperture only, (2) for point sources as the product of the square of the angular aperture times the square of the linear aperture.

II. The relative intensity of the image with respect to the field (contrast) for any celestial object not very near the horizon, will depend upon (1) the brightness of the sky at the time; (2) the efficiency of the image-forming lens as regards perfection of figure, nature and curvature of surfaces, &c.; and (3) upon the square of the linear aperture. If the objectives are good, the sky effect (1) and (3) are the most important.

III. For faint extended objects such as nebulæ, irresolvable star clouds, &c., in which we have to deal with the delineation of a surface rather than with individual points, this contrast can only be increased by decreasing the focal length. When the sky effect (1) and (3) is predominant it will vary inversely as the square of the latter quantity.

IV. For point sources the contrast can only be increased by increasing the angular aperture. Under the same conditions as in III, it will vary directly as the square of the angular aperture.

V. It is the degree of contrast, and not size of objective (except in so far as this latter influences the contrast) that determines the limiting magnitude of the faintest object that can be photographed. This limiting magnitude for stars depends therefore only on the angular aperture of the instrument, and for nebulæ only on the focal length.

VI. The time of exposure also depends very largely on the contrast between image and field, and not on the absolute intensity of the former. If the development could be suited in every case to the exposure, the time of the latter necessary to obtain any given degree of detail would vary inversely as the contrast within a wide range of photographic light action.

VII. In the problem of photographing the corona without an eclipse, the general conclusion in III is modified to some extent by the increased brightness and increased whiteness of the sky near the Sun, due probably to the presence of water vapour in the lower strata of our atmosphere. It seems, however, certain that a great advantage will result from using an image-forming lens of very small linear aperture and very short focal length for this purpose. Those who have used pin holes have been right in so far as a small linear aperture is concerned, but wrong in so far as they have placed the plate at a distance from the opening.

VIII. For nebular spectrographic work the integrating spectrograph (one in which the image-forming or condensing lens is of zero dimensions), is the best possible form for securing a maximum contrast.

IX. In stellar spectrographic as well as in stellar photographic work, the "condensing" (telescope) objective should have the largest possible angular aperture.*

X. As regards the influence of the character of the objective (II (2)) upon the illumination of the field, the refractor seems to have a decided advantage. The angular aperture of the latter should not, however, be greater than 1-5. To photograph the very faintest star (beyond 17th magnitude) a reflector of the largest possible angular aperture, 1-3 or 1-2 if possible, is the only instrument that can be used. For stellar spectrographic work, in which only a small field is required, the reflector of large angular aperture is greatly superior to the refractor.†

Yerkes Observatory:
1897 *June* 3.

A Note on Spider Lines. By F. L. O. Wadsworth, Astrophysicist, Yerkes Observatory.

(*Communicated by the Secretaries.*)

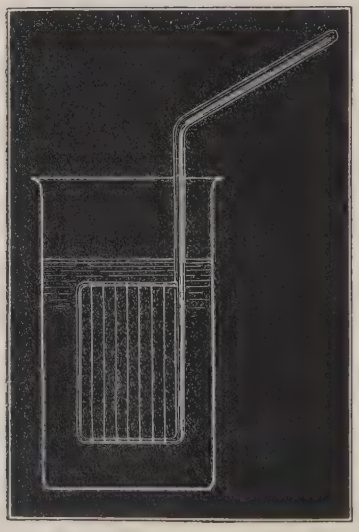
Apropos of the present discussion on the straightness of spider lines—see paper by Professor Turner in the last (April) number of the *Monthly Notices*—it may be well to point out that whether such an effect has ever been observed or not, it would be well to guard against its possible occurrence by using for the cross wires in our instruments something of a more reliable and permanent nature than the usual spider web. This, although very fine and strong, has so many and such grave disadvantages as to make one wonder how it has ever been tolerated as long as it has. Formerly perhaps it might have been said that there was no thoroughly satisfactory substitute; even the finest metal wires prepared by the so-called Wollaston process being too coarse, and silk having the same disadvantages as spider line itself in the way of hygroscopicity and semi-translucency. Now, however, the invention of the quartz fibre by Boys has provided us with something

* In connection with this point see also my paper, "General Conditions respecting the Design of Astronomical Spectroscopes," in the first number of the *Astrophysical Journal*, Jan. 1895, p. 52 (particularly pp. 70–71).

† For other advantages of the reflector see also paper by Professor Hale in a recent number of the *Astrophysical Journal* (Feb. 1897, p. 119), and various papers by the writer in the *Phil. Mag.* (vol. xxxviii. pp. 137 and 337), *Astronomy and Astrophysics* (vol. xiii. p. 835), and the *Astrophysical Journal* (vol. i. p. 232; vol. ii. p. 370; vol. iii. p. 169 [particularly pp. 182 and 183]; vol. iii. p. 347; &c.).

which is not only entirely unaffected by atmospheric changes, but also superior to spider line, and, in fact, to all other known fibres, in strength and fineness. The quartz fibres themselves are transparent, but this difficulty is removed, and the fibres made more easily visible for handling and mounting as well, by silvering them by any of the well-known silvering processes. When properly silvered they are perfectly opaque, even in the most strongly illuminated field.

I do not know who first proposed the use of silvered quartz fibres for this purpose, but their obvious advantages must have suggested their use for this and many similar purposes (such as fine plumb and level lines, &c.) to many others besides myself. I first used them for the cross wires of an observing telescope about five years ago, while a student at Clark University. It is hardly necessary to say that I have continued to use them ever since whenever it was possible to obtain or make them, and I know of many others whose experience has been equally satisfactory. Nevertheless, they do not yet seem to be as generally known and widely used as they ought to be. It is sometimes difficult to obtain sufficiently fine fibres from dealers, at least in this country ; but anyone can, after a little practice, make them for himself, the only really essential things being a good oxyhydrogen blow-pipe and a supply of reasonably pure hydrogen (coal gas is very unsatisfactory), and oxygen in pressure tanks. These requisites can now all be readily obtained from any good dealer in projection apparatus. Many plans for "shooting" the fibres have been proposed, but I have never found anything more simple or satisfactory than the original bow and arrow method of Pro-



Frame for silvering quartz fibres.

fessor Boys. See paper "On the Production Properties and Some Suggested Uses of the Finest Fibres," *Phil. Mag.* 1887 June.

For silvering them I prefer Brashear's method (described at length in a paper by the writer, "On Silvering Solutions and Silvering," *Astrophysical Journal*, vol. 1, page 252, 1895 March). To get a good coat it is, however, necessary to clean the fibre very carefully. For this purpose I have found it best to use in succession very hot solutions of strong nitric acid and strong potash, and then wash in several changes of clean distilled water before immersing in the silvering solution. In order to handle the fibres during these operations I have found it convenient to mount them on a small rectangular glass frame with a bent handle, similar to that shown in the figure. The ends of the fibres are secured to the upper and lower bars of the frame by fused (not dissolved) shellac, which resists well the action of the hot acid and alkali. In lifting the frame in and out of the solutions care should be taken to move it vertically up and down, otherwise the fine fibres will be broken by the surface tension of the liquid surface. On such a frame a dozen fibres may be silvered at once, and then set aside for use as desired.

Another great advantage which such silvered fibres would be likely to possess when used in bright wire micrometers, *i.e.* that of perfect and uniform illumination, was pointed out by Professor Burnham in the course of a recent conversation with the writer on this subject. To get the best results in such a case a cylindrical mirror should be placed behind the wire (on the side away from the source of illumination) with its axis parallel to the length of the wire.

Yerkes Observatory:
1897 *May* 29.

Notes on the Reduction of Stellar Photographs.

By Prof. Arthur A. Rambaut, M.A., D.Sc.

All who have closely followed the recent developments in the use of photography as a means of measuring the positions of stars must have been impressed with the great value of the methods proposed and advocated by Professor Turner in the several papers which he has published on the subject.

From the point of view taken up by him—which is in the main that of a partaker in the International Photographic Survey of the Heavens, although many of his results have a wider application and are largely useful in facilitating operations and increasing accuracy whatever the object of research may be—from this point of view it is difficult to see in what way much further simplification in the reduction of the plates is to be introduced.

No method for determining the relative positions on overlapping plates can be more simple or symmetrical than that proposed by him and adopted, I understand, at Greenwich, Oxford, and other places.

The secret of this remarkable simplification lies in the fact of his adhering strictly to rectangular coordinates, and avoiding carefully as long as it is possible the use of the old and time-honoured system of right ascension and declination, which the fact of our inhabiting a rotating body obliges us in the last extremity to use in some form or other when we desire to define the position of a celestial body in space.

I expect, however, that there are many workers in this line, who, like myself, are occupied in taking photographs under conditions different from those considered by Professor Turner, M. Loewy, and others, in connection with the Astrophotographic Catalogue, and who find the methods advocated in the *Bulletin du Comité International Permanent pour l'Exécution Photographique de la Carte du Ciel* are not exactly suitable to their particular requirements.

I have been recently engaged in considering the best mode of reducing the plates taken at Dunsink with our 15-inch reflecting telescope, in which we have a means of adjusting the plate with a fair degree of precision, and of thus keeping it very approximately "squared on" to the axis of the telescope, but the actual R.A. and declination of the point to which the telescope is directed is not known with any considerable degree of accuracy, and the plate is merely inserted in an ordinary slide without any particular care as to its orientation.

For work such as we are engaged upon, in which the objects photographed are scattered in all parts of the sky, the principle of the "rattachment" of plates, recommended by Loewy, Turner, and others, for the Astrographic Catalogue, and the elaborate mode of reduction which the former has founded upon it are clearly inadmissible, and some more applicable method must be sought.

In a case of this sort if we have a number of reference stars distributed fairly symmetrically over the plate so that the lines joining them would form a polygon inside of which lie the object or objects whose position it is desired to ascertain, it is possible to attain an accuracy at least equal to the mean accuracy in the positions of the reference stars, and any inaccuracy introduced by small errors in the adopted positions of the latter will not affect the determination of parallax or proper motion provided that the same set of reference stars is used on each occasion.

In what follows I suppose that the measures do not extend over a space of more than a square degree. This is only one quarter of the space covered by one of the plates of the Astrophotographic Catalogue, but this limitation will enable us to dispense with many of the smaller terms which complicate the reduction, and which only become sensible when distances greater than 2000″ have to be considered, while giving us as

wide a region around the centre of each plate in which to find the reference stars as is recommended for the reduction of the Catalogue plates.

In a case such as I have suggested the advantages of using rectangular coordinates can, I think, be best attained by having recourse at once to a system of what Professor Turner calls "standard" coordinates. These are coordinates referred to the projections of the hour circle and of the great circle at right angles to it passing through the origin.

But under the conditions I have supposed we do not know the R.A. and declination of the origin from which our measures are made on the plate, and if we assume an origin for the standard coordinates we do not know precisely the point on the plate representing it.

We have therefore two systems of rectangular axes, one in a tangent plane to the sphere at a point whose R.A. and declination we assume, and the other, that to which the measures are made, whose position in the plane of the plate is known. And what we require to find is a means of passing from one of these systems to the other.

This is afforded us by a comparison of the coordinates of the reference stars with regard to each of them.

Now let us denote by O the point of the plate which we take as the zero of measurement, while the centre of the plate (C) is defined as the point in which the normal to it from the centre of the objective, or from the centre of curvature of the mirror, cuts the plate. If we take a pair of axes passing through the centre of the sphere parallel to the axes of measurement during the exposure, then these two with the direction of the normal to the plate through C form a system of three mutually rectangular axes. Let the coordinates of the image (S) of a star σ referred to there be x, y, z.

If now we take another system of rectangular axes passing through the centre of the sphere of which the axes of x and y are parallel to our assumed standard axes, and if these make angles α, β, γ; α', β', γ'; $\alpha'', \beta'', \gamma''$ respectively with the previous system, and if the coordinates of the star σ referred to these be X, Y, Z we have

$$X : Y : Z = x\cos\alpha + y\cos\beta + z\cos\gamma : x\cos\alpha' + y\cos\beta' + z\cos\gamma' : x\cos\alpha'' + y\cos\beta'' + z\cos\gamma''$$

Therefore

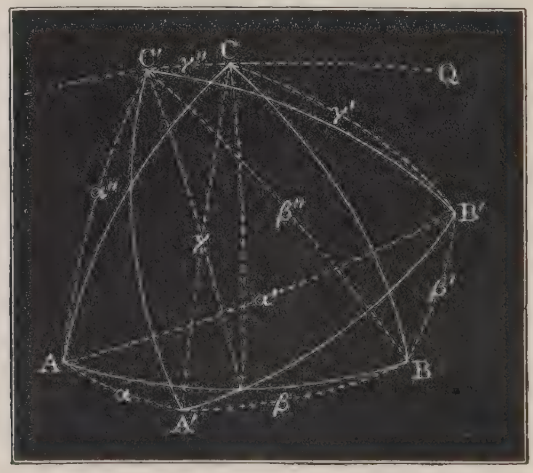
$$\left.\begin{aligned} \frac{X}{Z} &= \frac{\frac{x}{z}\frac{\cos\alpha}{\cos\gamma''} + \frac{y}{z}\frac{\cos\beta}{\cos\gamma''} + \frac{\cos\gamma}{\cos\gamma''}}{\frac{x}{z}\frac{\cos\alpha''}{\cos\gamma''} + \frac{y}{z}\frac{\cos\beta''}{\cos\gamma''} + 1} \\ \text{and} \\ \frac{Y}{Z} &= \frac{\frac{x}{z}\frac{\cos\alpha'}{\cos\gamma''} + \frac{y}{z}\frac{\cos\beta'}{\cos\gamma''} + \frac{\cos\gamma'}{\cos\gamma''}}{\frac{x}{z}\frac{\cos\alpha''}{\cos\gamma''} + \frac{y}{z}\frac{\cos\beta''}{\cos\gamma''} + 1} \end{aligned}\right\} \quad \ldots\ldots\ (1)$$

But $-Z$ is the radius of the sphere while $-z$ is the focal length of the telescope. Hence $-\frac{X}{Z}, -\frac{Y}{Z}$ are the standard coordinates (ξ and η) of the star expressed in radians, while $-\frac{x}{z}, -\frac{y}{z}$ are the coordinates of the star's image on the plate expressed in the same manner. Now we may put $-Z$ equal to unity, and if we denote $-\frac{1}{z}$ by λ (where λ is the value of a unit of the screw or scale with which the measures are made) we find

$$\left.\begin{aligned}\xi &= \frac{\lambda\,(ax+by)+c}{1-\lambda\,(kx+ly)}\\ \eta &= \frac{\lambda\,(dx+ey)+f}{1-\lambda\,(kx+ly)}\end{aligned}\right\} \quad \ldots\ldots\ldots \quad (7)$$

These are the expressions found by Professor Turner in his first paper on this subject (*Monthly Notices*, liv. 1) except that he supposes $\lambda=1$, but this method of deducing them has the distinct advantage of showing plainly the geometrical meaning of each of the quantities involved.

For instance, from equations (1) we can exhibit in a very symmetrical manner the relations between the coefficients a, b, c, &c., deduced by Professor Turner in his paper in the *Monthly Notices*, vol. lvii. 3. For this purpose we have recourse to the accompanying figure in which A, B, C, A′, B′, C′ are the points in which the two sets of axes intersect the sphere.



Then

$$AA' = \alpha, \quad BA' = \beta, \quad CA' = \gamma$$
$$AB' = \alpha', \quad BB' = \beta', \quad CB' = \gamma'$$

and

$$AC' = \alpha'', \; BC' = \beta'', \; CC' = \gamma''$$

and since C and C′ are the poles of AB and A′B′ respectively, therefore the angle APA′$=\gamma''$. Also the angle QCA is Professor Turner's α which I denote by p, and QC′A′ is Professor Turner's β which I denote by q.

Then since P is the pole of CC′ it follows that

$$\mathrm{AP}\,(=\mathrm{ACP})=p-\frac{\pi}{2},\ \mathrm{PB}=\pi-p$$

and

$$\mathrm{A'P}\,(=\mathrm{A'C'P})=q-\frac{\pi}{2},\ \mathrm{PB'}=\pi-q$$

Hence it follows at once that

$$\begin{aligned}
\cos\alpha &= \sin p\sin q+\cos p\cos q\cos\gamma''\\
\cos\beta &= -\cos p\sin q+\sin p\cos q\cos\gamma''\\
\cos\gamma &= \cos q\sin\gamma''\\
\cos\alpha' &= -\sin p\cos q+\cos p\sin q\cos\gamma''\\
\cos\beta' &= \cos p\cos q+\sin p\sin q\cos\gamma''\\
\cos\gamma' &= \sin q\sin\gamma''\\
\cos\alpha'' &= -\cos p\sin\gamma''\\
\cos\beta'' &= -\sin p\sin\gamma''
\end{aligned}$$

Hence,

$$\left.\begin{aligned}
a &= \frac{\cos\alpha}{\cos\gamma''} = \cos p\cos q+\sin p\sin q\sec\gamma''\\
b &= \frac{\cos\beta}{\cos\gamma''} = \sin p\cos q-\cos p\sin q\sec\gamma''\\
c &= -\frac{\cos\gamma}{\cos\gamma''} = -\cos q\tan\gamma''\\
d &= \frac{\cos\alpha'}{\cos\gamma''} = \cos p\sin q-\sin p\cos q\sec\gamma''\\
e &= \frac{\cos\beta'}{\cos\gamma''} = \sin p\sin q+\cos p\cos q\sec\gamma''\\
f &= -\frac{\cos\gamma'}{\cos\gamma''} = -\sin q\tan\gamma''\\
k &= \frac{\cos\alpha''}{\cos\gamma''} = -\cos p\tan\gamma''\\
l &= \frac{\cos\beta''}{\cos\gamma''} = -\sin p\tan\gamma''
\end{aligned}\right\}\quad\ldots\quad(3)$$

These are the relations found by Professor Turner. From the values of these quantities expressed in terms of the direction-cosines, we see that

$$a^2+b^2+c^2=d^2+e^2+f^2=1+k^2+l^2=\sec^2\gamma'',$$

relations which afford an important check on the accuracy of the computation.

From these equations, again, it appears that

$$c-k=(\cos p-\cos q)\tan\gamma''=-2\sin\tfrac{1}{2}(p-q)\sin\tfrac{1}{2}(p+q)\tan\gamma''$$

and

$$f-l=(\sin p-\sin q)\tan\gamma''=\quad 2\sin\tfrac{1}{2}(p-q)\cos\tfrac{1}{2}(p+q)\tan\gamma''$$

But $p-q$ can never exceed a, so that the differences $c-k$ and $f-l$ are of the order $a\gamma''$, and hence to a first approximation may be neglected.

If we denote by x', y' the coordinates of the image S of a star σ, measured parallel to rectangular axes, passing through any point O on the plate, and if x_0, y_0 are the coordinates of the centre C referred to the same axes, then the coordinates of S referred to parallel axes through C are $x=x'-x_0$, $y=y'-y_0$.

For the origin of standard coordinates, we may take any point of the sphere whatever, the right ascension and declination of which we denote by A and D, and we shall have equations (2) connecting ξ, η, with x, y.

But if γ'' is a small quantity, *i.e.*, if the centre of the plate is close to the origin of standard coordinates—a condition which it is easy to fulfil—these expressions may be very much simplified.

For if we substitute from equations (3), neglecting powers of γ'' and λx, λy, beyond the third, and denoting the angle $q-p$ by ϕ, we find,

$$\xi=\frac{\lambda(x\cos\phi-y\sin\phi)+\cos q\,.\,\gamma''+\frac{1}{2}\sin q\,.\,\lambda(x\sin p-y\cos p)\gamma''^2+\frac{1}{3}\cos q\,.\,\gamma''^3}{1+\lambda(x\cos p+y\sin p)(\gamma''+\frac{1}{3}\gamma''^3)}.$$

Now, if γ'' does not exceed $100''$, we may reject its cube and terms of the first order in x, y, when multiplied by its square, and of the second order multiplied by γ''.

To this order of approximation we find simply

$$\xi=\lambda(x\cos\phi-y\sin\phi)+\cos q\,.\,\gamma'',$$

while in a similar manner we obtain

$$\eta=\lambda(x\sin\phi+y\cos\phi)+\sin q\,.\,\gamma''.$$

Hence we see that $\cos q\,.\,\gamma''$ and $\sin q\,.\,\gamma''$ are the standard coordinates of the centre with regard to (A, D). Denoting them by ξ_0, η_0, we have finally,

$$\left.\begin{aligned}\xi-\xi_0&=\lambda[(x'-x_0)\cos\phi-(y'-y_0)\sin\phi]\\ \eta-\eta_0&=\lambda[(x'-x_0)\sin\phi+(y'-y_0)\cos\phi]\end{aligned}\right\}\quad\ldots\quad(4)$$

These are the ordinary formulæ of transformation from one pair of rectangular axes to another, and their applicability

depends on the fact that the assumption we have made as to the magnitude of γ'' is equivalent to assuming that the centre of the plate is so near the origin of standard coordinates, that the straight lines into which the latter project are sensibly at right angles to each other within the limits of the plate.

By means of the equations (4), we can, from the standard coordinates of two known stars, determine the values of λ and ϕ, whatever their magnitude may be, but in practice ϕ will always be a small angle, and λ will differ but little from unity. In this case assume

$$\lambda \cos \phi = 1 + \mu \cos \nu = 1 + P$$
$$\lambda \sin \phi = \mu \sin \nu = Q,$$

where P and Q are both small quantities. Then denoting by the subscript figures 1 and 2, the coordinates relating to the two reference stars, we have

$$(\xi_1 - \xi_2) - (x'_1 - x'_2) = \mu \cos \nu (x'_1 - x'_2) - \mu \sin \nu (y'_1 - y'_2)$$
$$(\eta_1 - \eta_2) - (y'_1 - y'_2) = \mu \sin \nu (x'_1 - x'_2) + \mu \cos \nu (y'_1 - y'_2).$$

Then assuming

$$\tan \psi_{12} = \frac{(\eta_1 - \eta_2) - (y'_1 - y'_2)}{(\xi_1 - \xi_2) - (x'_1 - x'_2)} \text{ and } \tan \theta_{12} = \frac{y'_1 - y'_2}{x'_1 - x'_2}$$

we have $\tan \psi_{12} = \tan (\nu + \theta_{12})$ or $\nu = \psi_{12} - \theta_{12}$.

Also

$$\sqrt{[(\xi_1 - \xi_2) - (x'_1 - x'_2)]^2 + [(\eta_1 - \eta_2) - (y'_1 - y'_2)]^2} = \mu \sqrt{(x'_1 - x'_2)^2 + (y'_1 - y'_2)^2},$$

whence

$$\mu = \frac{(\xi_1 - \xi_2) - (x'_1 - x'_2)}{x'_1 - x'_2} \cdot \frac{\cos \theta_{12}}{\cos \psi_{12}} = \frac{(\eta_1 - \eta_2) - (y'_1 - y'_2)}{y'_1 - y'_2} \frac{\sin \theta_{12}}{\sin \psi_{12}}.$$

Equations (4) may be written

$$\xi - x' - Px' + Qy' = \xi_0 - x_0 - Px_0 + Qy_0 = \bar{\xi}$$

and

$$\eta - y' - Qx' - Py' = \eta_0 - y_0 - Qx_0 - Py_0 = \bar{\eta}.$$

Substituting the values of P and Q in the left hand members, we find the values of $\bar{\xi}$, $\bar{\eta}$, which are the coordinates of O, referred to the standard axes through (A, D).

If the values of P and Q, as derived from different pairs of reference stars, are sensibly the same, so that the various values of $\bar{\xi}$ and $\bar{\eta}$ do not differ by more than the probable error in the coordinates of the stars will account for, we need go no further, but may assume P, Q, $\bar{\xi}$, $\bar{\eta}$ as the constants of the plate.

If, however, large differences occur in the values of P, Q, $\bar{\xi}$, $\bar{\eta}$, it follows that γ'' is not as small a quantity as is contemplated above, and it will be necessary to transform the standard axes to a point more nearly represented by the centre of the plate.

Of course, the quantities ξ, η, $\bar{\xi}$, $\bar{\eta}$ are *apparent* coordinates, and in deducing them from the known declinations and right ascension, the effects of refraction and aberration must be applied before they are compared with the measures.

The true values of these quantities are found from the formulæ

$$\left.\begin{aligned} \xi &= \frac{\cos\delta \sin(\alpha - A)}{\sin\delta \sin D + \cos\delta \cos D \cos(\alpha - A)} \\ \text{and} \\ \eta &= \frac{\sin\delta \cos D - \cos\delta \sin D \cos(\alpha - A)}{\sin\delta \sin D + \cos\delta \cos D \cos(\alpha - A)} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (5)$$

α and δ being the right ascension and declination of the star (compare Ball and Rambaut "On the Relative Positions of 223 Stars in the Cluster χ Persei." *Transactions of the Royal Irish Academy*, vol. xxx. pt. 4, p. 235).

These are the same expressions as those given by Professor Turner in the *Monthly Notices*, vol. liv., 1, p. 13.

In most cases the approximate formulæ deduced from these will be found more convenient. These are, denoting $\alpha - A$ and $\delta - D$ by $\Delta\alpha$ and $\Delta\delta$ respectively,

$$\xi = \cos D \,.\, \Delta\alpha - \sin 1'' \,.\, \sin D \,.\, \Delta\alpha \,.\, \Delta\delta - \frac{1}{2}\sin^2 1'' \,.\, \cos D \,.\, \left(\frac{1}{3} - \cos^2 D\right) \,.\, \Delta\alpha^3,$$

and

$$\eta = \Delta\delta + \frac{1}{4}\sin 1'' \,.\, \sin 2D \,.\, \Delta\alpha^2 + \frac{1}{3}\sin^2 1'' \,.\, \Delta\delta^3 + \frac{1}{2}\sin^2 1'' \,.\, \cos 2D \,.\, \Delta\alpha^2 \,.\, \Delta\delta.$$

For computing the coefficient of $\Delta\alpha^3$ in the first of these the following table for Log. $(\frac{1}{3} - \cos^2 D)$ is useful, three places of decimals being amply sufficient for any cases likely to occur in practice.

Table for Log $\left(\frac{1}{3} - \cos^2 D\right)$.

D.	Log $\left(\frac{1}{3} - \cos^2 D\right)$.		D.	Log $\left(\frac{1}{3} - \cos^2 D\right)$.		D.	Log $\left(\frac{1}{3} - \cos^2 D\right)$.		D.	Log $\left(\frac{1}{3} - \cos^2 D\right)$.	
°			°			°			°		
0	−9·824	0	23	−9·711	11	45	−9·222	48	68	+9·286	25
1	·824	1	24	·700	12	46	·174	54	69	·311	24
2	·823	1	25	·688	12	47	·120	62	70	·335	22
3	·822	1	26	·676	13	48	−9·058	71	71	·357	19
4	·821	2	27	·663	14	49	−8·987	85	72	·376	18
5	·819	2	28	·649	14	50	·902	105	73	·394	16
6	·817	3	29	·635	15	51	·797	137	74	·410	15
7	·814	3	30	·620	17	52	·660	201	75	·425	14
8	·811	3	31	·603	17	53	·459	376	76	·439	12
9	·808	4	32	·586	18	54	−8·083		77	·451	11
10	·804	5	33	·568	19	55	+7·633	681	78	·462	11
11	·799	4	34	·549	20	56	8·314	251	79	·473	9
12	·795	5	35	·529	22	57	·565	155	80	·482	8
13	·790	6	36	·507	23	58	·720	113	81	·490	7
14	·784	6	37	·484	25	59	·833	88	82	·497	6
15	·778	7	38	·459	27	60	·921	72	83	·503	5
16	·771	7	39	·432	28	61	8·993	60	84	·508	5
17	·764	7	40	·404	31	62	9·053	51	85	·513	4
18	·757	8	41	·373	33	63	·104	46	86	·517	2
19	·749	9	42	·340	36	64	·150	39	87	·519	2
20	·740	9	43	·304	39	65	·189	36	88	·521	1
21	·731	10	44	·265	43	66	·225	32	89	·522	1
22	·721	10	45	·222		67	·257	29	90	·523	
23	·711					68	·286				

The corresponding expressions for deducing $\Delta\alpha$ and $\Delta\delta$ from the values of ξ and η, are given in the above mentioned memoir by Ball and Rambaut, and are

$$\left.\begin{aligned} \Delta\alpha \cos D &= \xi + \sin 1'' \tan D \,.\, \xi\eta - \frac{1}{3}\sin^2 1'' \sec^2 D \,.\, \xi^3 + \sin^2 1'' \tan^2 D \,.\, \xi\eta^2 \\ \text{and} \\ \Delta\delta &= \eta - \frac{1}{2}\sin 1'' \tan D \,.\, \xi^2 - \frac{1}{2}\sin^2 1'' \sec^2 D \,.\, \xi^2\eta - \frac{1}{3}\sin^2 1'' \,.\, \eta^3. \end{aligned}\right\} \quad (6)$$

These formulæ contain all the terms which it is necessary to consider, provided that ξ and η do not exceed 2000″, and that D is less than 75°. This will include the vast majority of cases likely to occur in practice, but if it be desired to extend the measures much beyond half a degree from the centre and for

regions nearer the pole, higher terms must be considered or the exact formulæ employed.

The terms of the fourth order are given in a paper by Östen Bergstand, "Sur la Réduction des Mesures Micrométriques des Clichés Photographiques Stellaires" (*Öfversigt af Kongl. Vetenskaps Akademiens Förhandlingar*, 1896, No. 7), while those of the fourth and fifth orders will be found in a paper by Dr. Jacoby, on "The Reduction of Stellar Photographs," contained in the *Contributions from the Observatory of Columbia University, New York*, No. 10.

In the latter paper the author prefers to employ the quantity $\frac{1}{15}\xi$ sec D instead of ξ, which renders his formulæ somewhat cumbersome and more unattractive than is necessary, but they can be easily transformed so as to express $\Delta\alpha$ and $\Delta\delta$ in terms of ξ and η directly.

Refraction.

In the majority of cases likely to occur in practice the formulæ for the refraction, deduced by Professor Turner in his paper published in the *Monthly Notices*, vol. liv. No. 1, containing only terms of the first order in ξ and η, will be amply sufficient. For the limits at which the terms of the second order and those depending on $\frac{d\,\beta_0}{d\,\zeta}$ and $\beta_0{}^2$ become appreciable, compare a paper by the present author entitled "The Corrections for Refraction to Measures of Stellar Photographs," in the *Astronomische Nachrichten*, No. 3125.*

Professor Turner's formulæ are

$$d\xi = -\beta_0\,(1+X^2)\,\xi - \beta_0 XY\,.\,\eta$$

and

$$d\eta = -\beta_0 XY\,.\,\xi - \beta_0\,(1+Y^2)\,.\,\eta$$

in which X, Y denote the coordinates of the projection of the zenith on the plane containing the standard axis, and β_0 is the coefficient of refraction, corresponding to the point where these standard axes intersect at the middle of the exposure.

The simplicity and symmetry of these expressions must recommend them to every one who is familiar with the more cumbrous formulæ, which become necessary when the relative positions of stars are defined by means of their differences of R.A. and declination.

But although, as they stand, they are all that could be desired from an analytical point of view, yet as Professor Turner leaves them they are not as convenient as they might be for numerical computation.

* See also Kapteyn, *Bulletin du Comité International Permanent*, etc., Tome III., and Bergstrand, *Öfversigt af Kongl. Vetenskaps Akademiens Förhandlingar*, 1897, No. 2.

They may, however, be very readily transformed into a more convenient form. For since in the case of the zenith—for which $\alpha=\theta$, the sidereal time at the middle of the exposure, and $\delta=\phi$—we have, see equations (5).

$$X=\frac{\cos\phi\sin(\theta-A)}{\sin\phi\sin D+\cos\phi\cos D\cos(\theta-A)}$$

and

$$Y=\frac{\sin\phi\cos D-\cos\phi\sin D\cos(\theta-A)}{\sin\phi\sin D+\cos\phi\cos D\cos(\theta-A)}$$

if we take m and n such that

$$\cos n=\cos\phi\sin(\theta-A)$$
$$\sin n\sin m=\cos\phi\cos(\theta-A)$$

and

$$\sin n\cos m=\sin\phi$$

we find

$$X=\frac{\cos n}{\sin n\sin(m+D)}\text{ and }Y=\frac{\sin n\cos(m+D)}{\sin n\sin(m+D)}.$$

Hence,

$$1+X^2=\frac{1-\sin^2 n\cos^2(m+D)}{\sin^2 n\sin^2(m+D)}$$

$$XY=\frac{\sin n\cos n\cos(m+D)}{\sin^2 n\sin^2(m+D)}$$

$$1+Y^2=\frac{\sin^2 n}{\sin^2 n\sin^2(m+D)}$$

But if ζ denote the zenith distance

$$\cos\zeta=\sin n\sin(m+D),$$

—a quantity we have to compute in order to obtain β_0—and if we take

$$\sin n\cos(m+D)=\cos n'$$

we have

$$1+X^2=\frac{\sin^2 n'}{\cos^2\zeta},\quad XY=\frac{\cos n'\cos n}{\cos^2\zeta},\quad 1+Y^2=\frac{\sin^2 n}{\cos^2\zeta}.$$

But m and n, given by the equations

$$\tan m=\cot\phi\cos(\theta-A)$$

and

$$\cot n=\sin m\tan(\theta-A),$$

are the quantities ordinarily used for computing refraction and parallactic angle and are taken directly, with the hour angle as argument, from tables which should be computed for each observatory.

We have then merely to compute

$$\cos \zeta = \sin n \sin (m + \mathrm{D}),$$

whence we find ζ, and therefore β_0, and

$$\cos n' = \sin n \cos (m + \mathrm{D})$$

whence we find sin n' and the refraction formulæ as applied to standard rectangular coordinates become simply

$$d\xi = -\frac{\beta_0}{\cos^2 \zeta}[\sin^2 n' \,.\, \xi + \cos n' \cos n \,.\, \eta]$$

and

$$d\eta = -\frac{\beta_0}{\cos^2 \zeta}[\cos n' \cos n \,.\, \xi + \sin^2 n \,.\, \eta],$$

while equations of the same form, but with a different value of β_0, will give the correction to the apparent ξ and η as deduced from the measures to reduce them to true standard coordinates before transforming into $\Delta\alpha$ and $\Delta\delta$ by means of equations (6).

Aberration.

In the paper referred to above, Professor Turner finds for the effect of aberration the still simpler formulæ

$$d\xi = -\frac{\gamma_0}{\sqrt{1 + \mathrm{X}^2 + \mathrm{Y}^2}} \,.\, \xi \qquad \text{and} \qquad d\eta = -\frac{\gamma_0}{\sqrt{1 + \mathrm{X}^2 + \mathrm{Y}^2}} \,.\, \eta$$

in which γ_0 is the constant of aberration, and (X, Y) is the projection of the "Earth's Way" on the plane of the plate.

As in the case of the refraction formulæ, these expressions may be readily transformed so as to be more convenient for numerical computation.

By the use of Bessel's Day Numbers from the Ephemeris combined with the "Tables for facilitating the Computation of Star Constants," compiled by the late Dr. Stone, the computation of this correction assumes an exceedingly simple form.

For we have

$$\frac{1}{\sqrt{1 + \mathrm{X}^2 + \mathrm{Y}^2}} = \cos \theta_0$$

where θ_0 denotes the angle between the "Earth's Way" and the point of the sky represented by the centre of the plate.

Hence,

$$\frac{-\gamma_0}{\sqrt{1 + \mathrm{X}^2 + \mathrm{Y}^2}} = -\gamma_0 \cos \theta_0 = -\gamma_0[\sin \bar{\delta} \sin \mathrm{D} + \cos \bar{\delta} \cos \mathrm{D} \cos (\bar{\alpha} - \mathrm{A})]$$

in which $\bar{\alpha}$ and $\bar{\delta}$ denote the R.A. and declination of the "Earth's

Way." But, $\odot$ denoting the Sun's longitude and ϵ the obliquity of the ecliptic, we have

$$\cos \bar{\alpha} \cos \bar{\delta} = \sin \odot$$
$$\sin \bar{\alpha} \cos \bar{\delta} = -\cos \odot \cos \epsilon$$
$$\sin \bar{\delta} = -\cos \odot \sin \epsilon$$

Hence,

$$-\gamma_0 \cos \theta_0 = \gamma_0[\cos \odot \sin \epsilon \sin D - \sin \odot \cos D \cos A + \cos \odot \cos \epsilon \cos D \sin A].$$

Writing D′ for $\frac{\pi}{2} - D$ this becomes

$$-\gamma_0 \cos \theta_0 = -\gamma_0[\cos \odot \cos \epsilon\{-\cos D' \tan \epsilon - \sin D' \sin A\} + \sin \odot (-\sin D' \cos A)]$$

But

$$\left.\begin{aligned} -\gamma_0 \cos \odot \cos \epsilon &= A \\ -\gamma_0 \sin \odot \quad\ \ &= B \end{aligned}\right\} \text{from the Ephemeris}$$

And—without having regard to signs—

$$\sin D' \sin A = a^1, \ \cos D' \tan \epsilon = sa^1, \text{ and } \sin D' \cos A = b^1,$$

a^1, sa^1, b^1 being the quantities denoted by the same symbols in Dr. Stone's tables corresponding to the declination D′.

We thus find that

$$d\xi = \{A(a^1 + sa^1) + B.b^1\} \sin 1''. \quad \xi$$

with a similar expression for $d\eta$.

With regard to the signs of a^1, sa^1, and b^1, it is easy to see that, adopting the convention that ξ increases in the direction of increasing R.A. and η in that of increasing declination, they may be taken from the following table :—

Signs of the Constants.

North of Equator.			R.A.	South of Equator.		
a^1.	sa^1.	b^1.	h h	a^1.	sa^1.	b^1.
−	−	+	0 to 6	−	+	+
−	−	−	6 „ 12	−	+	−
+	−	−	12 „ 18	+	+	−
+	−	+	18 „ 24	+	+	+

It is not necessary to consider the corrections for Precession and Nutation. For since these are merely changes affecting the positions of the circles from which R.A. and Declination are measured, and have no effect on the relative positions of the stars themselves, they need not be taken into account until we convert the measured rectangular coordinates into differences of R.A. and declination, and then the corrections can best be computed by means of the ordinary tables.

The only remaining disturbance affecting the position of a star which it is necessary to consider is the annual displacement due to

Parallax.

This may be at once deduced from the formulæ for the aberration given by Professor Turner in note iii of his paper in the *Monthly Notices*, vol. liv. No. 1. If we write π for γ_0 (π being the annual parallax), and let X, Y be the coordinates of the projection of the Sun on the plate, then the parallactic displacement is

$$d\xi = \pi \frac{X - \xi}{\sqrt{1 + X^2 + Y^2}}$$

and

$$d\eta = \pi \frac{Y - \eta}{\sqrt{1 + X^2 + Y^2}}$$

in which the eccentricity of the Earth's orbit is neglected.

In these expressions we may reject ξ and η and we get as the effect of parallax

$$d\xi = \frac{\pi X}{\sqrt{1 + X^2 + Y^2}}$$

and

$$d\eta = \frac{\pi Y}{\sqrt{1 + X^2 + Y^2}}$$

Now

$$\frac{1}{\sqrt{1 + X^2 + Y^2}} = \cos \theta_0$$

θ_0 being in this case the angle between the direction of the Sun and that of the centre of the plate. Hence, α and δ being the Sun's R. A. and declination

$$\frac{1}{\sqrt{1 + X^2 + Y^2}} = \sin \delta \sin D + \cos \delta \cos D \cos (\alpha - A)$$

But

$$X = \frac{\cos \delta \sin (\alpha - A)}{\sin \delta \sin D + \cos \delta \cos D \cos (\alpha - A)}$$

and

$$Y = \frac{\sin \delta \cos D - \cos \delta \sin D \cos (\alpha - A)}{\sin \delta \sin D + \cos \delta \cos D \cos (\alpha - A)}$$

Hence we have simply

$$d\xi = \pi \cos \delta \sin (\alpha - A)$$

and

$$d\eta = \pi (\sin \delta \cos D + \cos \delta \sin D \cos [\alpha - A])$$

or, substituting for α and δ in terms of the Sun's longitude and the obliquity of the ecliptic,

$$d\zeta = -\pi(\cos \odot \sin A - \sin \odot \cos \epsilon \cos A)$$

and

$$d\eta = -\pi(\cos \epsilon \sin A \sin D - \sin \epsilon \cos D) \sin \odot - \pi \cos \odot \sin D \cos A$$

in which $d\eta$ is the well known expression for the annual parallax in declination, while $d\xi$ is the parallax in R.A. multiplied by cos D.

Dunsink Observatory, Co. Dublin:
1897 *May* 28.

Photographic Observations of Comet b 1896.

By Professor Arthur A. Rambaut, M.A., Sc.D.

On the nights of 1896 April 29 and 30, May 8 and 11, photographs were taken of Comet *b* 1896 with the 15-inch reflecting telescope of the Dunsink Observatory. After that date, owing to increasing twilight and diminishing brightness of the comet, it could not be found in the 5-inch guiding telescope.

An exposure of ten minutes was given on each occasion, and except in the case of plate *c* on April 30 the movement was adjusted to sidereal rate, and the image of the comet accordingly appears as an elongated patch with ill-defined edges.

The middle point of the axis of this patch was taken as the position of the comet corresponding to the mean time of exposure.

The measures were made in the Troughton and Simms microscope, described in *Transactions of the Royal Irish Academy*, vol. xxx. part iv.

The microscope is provided with two screws, very approximately at right angles to each other, and readings were taken on both screws simultaneously.

The plate was then turned through exactly 90° by the position circle, and another set of measures taken. We thus obtained the rectangular coordinates independently on each screw, and these were separately reduced by the formulæ given in the preceding paper, the mean of the two being taken as the position of the comet given in the table below.

There is not the material for determining the probable error of these positions in the usual way, but an estimate of their precision may be gathered from the following results.

From the measures of +51°, 686 on plate *b*, which were

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made at the same time as the others, but were not used in the reduction, I find

+51°, 686.	R.A. 1896·0.	Decl. 1896·0.
	h m s	° ′ ″
First Exposure	3 2 54·92	+51 58 30·8
Second Exposure	54·89	31·1

Compare No. 12 in the list of comparison stars below.

On plate *e*, too, the position of +65°, 217 was measured simultaneously with that of the comet, with the following results :—

+65°, 217.	R.A. 1896·0.	Decl. 1896·0.
	h m s	° ′ ″
First Exposure	1 51 12·62	+65 59 8·8
Second Exposure	12·60	8·6

It only remains to add that the position of the comet being by the mode of reduction referred to the mean positions of the stars for the beginning of the year, I have not thought it necessary to give its apparent R.A. and decl. as is usually done.

Observations of Comet b 1896.

Date.	Plate.	G.M.T.	Local Sidereal Time.	Mean R.A. of Comet.	Log ($p \times \Delta$).	Mean Decl. of Comet.	Log ($q \times \Delta$).	Reference Stars.
1896.		h m s	h m s	h m s		° ′ ″		
April 29	*a*	10 10 25	12 18 30	3 6 1·90	−9·5615	+50 0 47·9	−0·8761	14, 15, 16, 17.
30	*b*	10 9 35	12 21 36	3 1 9·59	−9·5616	+51 53 54·3	−0·8759	9, 10, 11, 13.
	b	10 23 48	12 35 51	3 1 6·70	−9·5272	+51 54 54·7	−0·8849	9, 10, 11, 13.
	c	10 38 56	12 51 2	3 1 5·72	−9·4853	+51 56 7·3	−0·8936	9, 10, 11, 12.
May 8	*d*	13 20 24	16 4 29	2 12 12·44	+9·5608	+63 2 31·0	−0·8648	5, 6, 7.
	d	13 34 54	16 19 1	2 12 8·38	+9·6094	+63 2 53·0	−0·8558	6, 7, 8.
11	*e*	12 47 31	15 43 20	1 51 30·25	+9·5999	+65 36 7·5	−0·8534	1, 2, 3, 4.
	e	13 2 11	15 58 3	1 51 26·09	+9·6489	+65 36 35·1	−0·8440	1, 2, 3, 4.

On plates *b*, *d* and *e* two exposures were made, and each exposure reduced separately.

In the case of *c*, the comet itself was kept on the cross-wires, and the stars allowed to trail on the plate. In the other cases, the guiding was done by the aid of one of the comparison stars.

On each occasion the comet appeared sensibly circular, with slight condensation towards the centre.

In the computation of $\mathrm{Log}(p \times \Delta)$ and $\mathrm{Log}(q \times \Delta)$ the adopted value of Sun's mean horizontal parallax is $8''{\cdot}80$.

The meridian-circle observations of the comparison stars were made by Mr. Chas. Martin.

Adopted Places of Comparison Stars, as determined by observations with the Meridian Circle at Dunsink.

No.	B. D.	Mag.	Mean R.A. 1896·0.	Mean Decl. 1896·0.	No. of Obs.	
			h m s	° ′ ″		
1	+65°, 214	9·5	1 49 2·96	+65 15 44·1	1	
2	+65, 216	8·6	1 50 7·23	+65 56 52·2	2	
3	+65, 218	9·4	1 53 22·11	+66 0 41·2	2	
4	+65, 219	9·5	1 53 47·78	+65 12 52·5	2	
5	+62, 369	7·0	2 8 7·14	+62 45 2·5	2	
6	+63, 319	9·3	2 11 49·52	+63 11 56·8	2	
7	+62, 379	9·0	2 12 40·83	+62 52 19·2	2	
8	+63, 327	8·6	2 15 35·77	+63 15 13·2	2	
9	+51, 678	8·8	3 0 1·08	+51 55 47·8	2	
10	+51, 680	9·4	3 0 29·83	+52 2 53·3	2	
11	+51, 683	9·0	3 1 37·09	+51 42 41·3	2	
12	+51, 686	9·1	3 2 55·03	+51 58 30·9	1	
13	+51, 689	7·3	3 4 13·03	+51 46 8·5	2	
14	+49, 871	8·3	3 4 49·76	+49 52 10·4	2	
15	+49, 873	9·2	3 5 46·24	+50 3 44·7	2	
16	+49, 877	8·7	3 6 41·04	+49 59 23·4	1	
17	+49, 885	9·0	3 8 9·13	+49 43 38·3	2	S.p. of two.

Dunsink Observatory, Co. Dublin:
1897 May 28.

Elements of Comet Perrine (f.) 1896 November 2.

By C. J. Merfield.

The following elements are based on observations taken by Mr. John Tebbutt, of Windsor, N.S.W., and kindly communicated to the writer some few days ago.

The selected places, after reducing the time to the meridian

of Greenwich and correcting for aberration, also applying the parallax corrections to α and δ, are as follows :—

	t	α	δ
	d	h m s	° ′ ″
Mar.	5·24871	19 45 0·19	−31 31 58·78
	10·25059	19 42 47·56	−35 22 55·29
	15·24568	19 39 16·54	−39 59 8·32

Adopting the ecliptic as the fundamental plane, these places referred to the mean equinox of 1897 are

Day of year 1897.	λ	β
	° ′ ″	° ′ ″
63·24871	292 30 56·62	−10 6 37·82
68·25059	291 21 2·65	−13 48 56·77
73·24568	289 47 29·75	−18 13 26·36

In the computation the complete expression for the ratio M of the curtate distances for the first and third observations was employed, the radii vectores and anomalies of the comet for the three fundamental positions being derived from the elements by Dr. Knopf, published in the *Astronomische Nachrichten* 3394. The value of log. M thus derived being 9·9093881, this required no correction after the calculation was completed, the various checks being satisfied. The logarithm of the curtate distance for the first observation being 0·1404402.

These values gave the following system of parabolic elements :—

$T = 1897$ Feb. 8·168615 Greenwich M.T.

$$\left.\begin{aligned} \omega &= 172^\circ\ 25'\ 1''{\cdot}14 \\ \Omega &= 86\ 29\ 10{\cdot}82 \\ i &= 146\ 11\ 3{\cdot}78 \end{aligned}\right\} \text{M.E. } 1897$$

$\log q = 0{\cdot}0263786$

Middle place.

$c - 0$

$\cos\beta'\Delta\lambda' = 6''{\cdot}7. \qquad \Delta\beta' = 1'{\cdot}1.$

There is an error in the elements published in the *Astronomische Nachrichten* 3394, the longitude of the ascending node should read 86° 27′ 51″·1 instead of 86° 17′ 51″·1.

Sydney : 1897 *April* 10.

The Expected Shower of Leonids in 1897. By W. F. Denning.

The return of the *Leonids* in November next, though it may fairly be expected to afford a pretty abundant display, will occur in the presence of a gibbous Moon and thus lose much of the conspicuous character which it may quite possibly have exhibited in a dark sky. At midnight following November 14 the Moon will be twenty days old and therefore nearing her last quarter (November 17, $2^h\ 2^m$), but her light will be sufficiently strong to overpower many of the smaller meteors and to moderate the apparent brilliancy of the larger ones.

In November we shall be placed fully two years before a maximum return is expected, and the density of that part of the stream so far in front of its associated comet is not likely to be considerable, but we may get a shower of perhaps fifty or sixty *Leonids* per hour for one observer on the night following November 14. It must be remembered that the radiant does not rise on that date until 10^h15^m P.M. and passes the meridian at 6^h30^m on the following morning. Observations are not, therefore, likely to be very productive until after midnight in consequence of the unfavourable position of the radiant.

In 1831 (two years preceding the great American shower of 1833) a French naval officer, Captain Berard, commanding the brig, *Le Loiret*, reported to M. Arago that "on November 13, at four in the morning, the sky being perfectly cloudless and a copious dew falling, he saw a number of shooting stars and luminous meteors of great dimensions. During upwards of three hours more than two per minute were seen." Dr. Wright, of Ohio, U.S.A., also witnessed an abundant fall of meteors on the same morning. In 1864 (two years before the fine European display of 1866) a star-shower of considerable strength was observed from the s.s. *Ellora*, off Malta, on the morning of November 13, for according to the report of those on board "there was a brilliant display of meteors all through the watch."

Seeing, therefore, that marked showers presented themselves in 1831 and 1864, we appear justified in expecting a tolerably active display in 1897. By a comparison of the observations made of the *Leonids* in 1895 and 1896 (*Monthly Notices*, lvi. 255, and lvii. 277), it will be noticed that a marked increase occurred in the latter year. A relatively far greater increase may be expected in the coming November, though the richest part of the stream will probably envelope the earth during daylight in England on November 14 and pass unseen.

From a comparison of the "brilliant showers" which occurred in the years 1865, 1866, 1867, and 1868, Mr. B. V. Marsh, of Philadelphia, U.S.A., concluded that there probably existed three concentric meteor-groups. "The densest or middle part of the meteoric dream through which the earth passed in 1866 (observed in Europe) and 1867 (observed in America)

appears to be flanked on either side by lateral and somewhat less dense and wider meteoric currents, at a distance from the orbit of the main stream, which the earth crossed in about twelve hours on the mornings of 1865 November 13, and 1867 November 15." (*B.A. Report*, 1869, p. 302.)

Assuming the node to advance along the ecliptic at the average rate, shown by a comparison with the times of old observations, of fifty-seven minutes of arc, with reference to a mean moveable equinox, during one period of revolution, then the stream recurs six and a half hours later in an ordinary year, and seventeen and a half hours earlier in leap year. Thus it is only the half hour which progressively accumulates and with leap year dropped once in 3 centuries out of 4, as in 1700, 1800, and 1900, throws forward the date of the shower's recurrence by about 3 days in a century.*

The central meteoric group observed in 1866 and 1867 is not likely to be visible in England this year, as the earth will be involved with it at about noon on November 14, but the other two streams ought to furnish many meteors on the preceding and following mornings. If the motion of the node of the meteoric orbit has continued its previous rate since 1866 then the three groups, conjectured to exist by Mr. Marsh, ought to be centrally passed by the earth at the following times :—

Group		h.	Duration.		From obs. in
Preceding	Nov. 13	$9\frac{1}{4}$	12 hours	1865, Nov. 12,	America
Central	14	0	$4\frac{1}{2}$ „	1866–7, „ 13,	Europe and America
Following	14	15	$13\frac{1}{2}$ „	1867–8, „ 14,	China and America

The earth's longitude on November 14 at noon is 52° 25′.

With regard to this peculiarity of construction it must be confessed that it rests on somewhat slender evidence, especially in respect to the sparse intervals supposed to outlie the central group. The observations on which the theory depends were only those furnished by five showers at the last epochal return, and it is hoped that during the next seven years a large number of new data may be obtained, at places differing widely in longitude, to assist in the solution of this problem. Under more favourable circumstances the voids near the central current may be found less definite than supposed, but there can be no doubt of the shower's rich sustenance during a period of more than forty-two hours. In meteoric astronomy it is sometimes the unexpected which happens, so that, without placing implicit reliance on the exact times of occurrence of predicted phenomena, observers should watch carefully throughout those nights which offer the best general prospect of success.

In 1898 and 1901 there will be no moonlight to interfere with the brilliant aspect of the shower, in 1900 the Moon will be half illuminated, and in 1899 and 1902 our satellite will be near the full. In England we should probably have witnessed the most brilliant

* See an interesting letter and diagram, explanatory of this part of the subject, in *Nature*, 1896 December 24, by Professor A. S. Herschel.

display of all in 1899, as it is due at about 1 A.M. on November 15, but the flood of moonlight over the sky will operate most unfavourably upon the visible nature of the phenomenon. In 1900 the circumstances are more to our advantage, and an unusually fine exhibition seems likely to occur just before sunrise on November 15, but the sky will be rather light in the presence of a moon at her last quarter.

One of the features which ought to be ascertained at every return is the time of apparent maximum of the shower. In deducing this the height of the radiant ought perhaps to be considered, for a radiant near the zenith will apparently yield more meteors than a stronger system with its radiant at a low altitude. In fact, the visible strength of a radiant depends largely, as pointed out by Lieut.-Colonel Tupman some years ago, upon its zenithal distance. The hourly number of meteors, for a radiant at the zenith, may be computed by multiplying the number actually counted in an hour by the secant of the zenith distance of the radiant. The number so derived will give the number of meteors which would have diverged from the radiant had it occupied a place in the zenith.

Another circumstance affecting the hourly number of *Leonids*, but chiefly during a comparatively weak display, is that there are many other showers of streak-leaving meteors in and surrounding *Leo* at the middle of November. Some observers count all the meteors shooting from the general direction of *Leo* as *Leonids*, and thus the actual number of the latter is exaggerated. Allowance should always be made for these minor streams. A list of their radiants appears in *Popular Astronomy*, 1894 March, and *The Observatory*, 1897 January. The principal of them are situated near δ and ι *Cancri*, μ, and ξ *Ursæ Majoris*, and π, λ, τ and β *Leonis*.

The real paths of several interesting *Leonids* and *Cancrids* were determined, in 1896 November, as the result of simultaneous observations at various stations. If a few additional observers co-operated to gather similar materials during the ensuing return a large number of meteors might be secured, and not only *Leonids* but members of the numerous contemporary showers such as *Taurids*, *Perseids*, *Aurigids*, *Cancrids*, *Ursids*, &c., might have their heights, radiants, &c., accurately attributed. It is proposed to make a special effort in this direction and to arrange for systematic watching during several hours on the mornings of November 14, 15, and 16 next. If observers in the N. of England looked to South, while those in the S. of England looked to North, and those in the S.E. and S.W. districts directed their observation to the north-west and north-east respectively, and recorded the apparent flights of the most conspicuous meteors which appeared, a plentiful collection of doubly observed objects would sure to be discovered amongst the observations.

Bristol: 1897 *June* 7.

Ephemeris for Physical Observations of the Moon, 1897 *September to* 1898 *April*. By A. Marth.

Greenwich Noon.	Selenographical Colong. of the Sun.	Selenographical Lat. of the Sun.	Geocentric Libration Sel. Long. of the Earth.	Geocentric Libration Lat. of the Earth.	Combined Amount.	Direction.
1897.	°	°	°	°	°	°
Sept. 4	4·80	−0·93	+2·45	+4·84	5·42	333·2
5	17·00	0·95	3·29	3·56	4·85	317·3
6	29·19	0·98	3·98	2·09	4·49	297·8
7	41·38	1·00	4·49	+0·51	4·52	276·5
8	53·56	1·02	4·82	−1·07	4·94	257·5
9	65·74	1·05	4·98	2·57	5·60	242·7
10	77·91	1·07	+4·95	3·91	6·30	231·6
11	90·08	1·09	4·71	5·02	6·88	223·1
12	102·25	1·11	4·26	5·86	7·24	215·9
13	114·42	−1·13	3·59	6·40	7·33	209·2
14	126·60	1·15	2·71	6·63	7·16	202·1
15	138·78	1·17	1·65	6 56	6·77	194·1
16	150·97	1·19	+0·45	−6·21	6·23	184·1
17	163·16	1·20	−0·85	5·59	5·66	171·4
18	175·35	1·22	2·16	4·73	5·20	155·5
19	187·55	1·23	3·42	3·66	5·01	137·0
20	199·75	1·25	4·53	2·42	5·13	118·1
21	211·96	1·26	5·41	−1·03	5·50	100·9
22	224·17	−1·27	−5·97	+0·43	5·98	85·9
Oct. 2	346·36	−1·39	+4·36	+3·63	5·67	309·9
3	358·55	1·41	5·12	2·20	5·57	293·3
4	10·74	1·42	5·60	+0·66	5·63	276·7
5	22·92	1·43	5 81	−0·89	5·88	261·3
6	35 09	1·44	5·81	2·36	6·26	247·9
7	47·26	1·46	+5·60	−3·68	6·74	236·6
8	59·42	1·47	5·23	4·79	7·09	227·4
9	71·58	1·48	4·70	5·65	7 34	219·6
10	83·74	1·49	4·01	6·22	7·40	212·6
11	95·90	−1·49	3·16	6·50	7·22	205·8
12	108·05	1·50	2·15	6·48	6·83	198·3
13	120·21	1·51	+1·01	−6·17	6·25	189·2
14	132·37	1·51	−0·26	5·59	5·59	177·4
15	144·54	1·52	1·60	4·77	5·03	161·5
16	156·71	1·52	2·96	3·75	4·78	141·8

Greenwich Noon.	Selenographical Colong. of the Sun.	Lat.	Geocentric Libration Sel. Long. of the Earth.	Lat.	Combined Amount.	Direction.
1897.	°	°	°	°	°	°
Oct. 17	168·88	1·52	4·27	2·57	4·91	121·0
18	181·06	1·52	5·44	−1·24	5·58	102·8
19	193·24	1·52	6·38	+0·17	6·38	88·5
20	205·43	−1·52	−6·99	+1·60	7·17	77·1
Nov. 1	351·85	−1·51	+6·90	−1·41	7·05	258·5
2	4·02	1·51	7·04	2·84	7·59	248·0
3	16·19	1·51	6·87	4·09	7·99	239·1
4	28·35	1·51	6·45	5·12	8·23	231·4
5	40·50	1·50	5·84	5·87	8·29	224·7
6	52·65	1·49	5·05	6·34	8·10	218·4
7	64·80	1·49	4·13	6·52	7·72	212·2
8	76·94	1·48	3·09	6·41	7·11	205·6
9	89·08	1·47	1·95	6·20	6·49	197·4
10	101·22	−1·46	+0·70	5·64	5·68	187·1
11	113·37	1·45	−0·61	4·83	5·87	172·8
12	125·51	1·44	1·98	3·82	4·30	152·7
13	137·66	1·43	3·34	2·64	4·26	128·3
14	149·81	1·42	4·68	−1·33	4·86	105·9
15	161·96	1·40	5·84	+0·05	5·84	89·5
16	174·12	1·39	6·82	1·45	6·97	77·9
17	186·29	1·37	7·49	2·82	8·00	69·3
18	198·46	1·36	7·80	4·09	8·80	62·2
19	201·64	−1·34	−7·61	+5·18	9·20	55·6
Nov. 29	332·56	−1·21	+7·42	−2·18	7·73	253·6
30	344·73	1·19	7·66	3·59	8·45	244·8
Dec. 1	356 89	1·18	7·52	4·76	8·89	237·5
2	9·05	1·16	7·06	5·67	9·05	231 1
3	21·20	1·14	6·34	6·29	8·92	225·0
4	33·35	−1·12	+5·41	−6·61	8·53	219·1
5	45·49	1·10	4·33	6·63	7·92	213·0
6	57·63	1·08	3·15	6·36	7·09	206·2
7	69·77	1·06	1·88	5·81	6·11	197·9
8	81·90	1·04	+0·57	−5·02	5·05	186·5
9	94·03	−1·01	−0·77	4·00	4·08	169·2
10	106·16	0·99	2·10	2 81	3·51	143·3
11	118·29	0·96	3·39	1·49	3·71	113·8
12	130·43	0·94	4·61	−0·10	4·61	91·2

Greenwich Noon.	Selenographical Colong. of the Sun.	Lat.	Geocentric Libration Sel. Long. of the Earth.	Lat.	Combined Amount.	Direc- tion.
1897.	°	°	°	°	°	°
Dec. 13	142·57	−0·91	−5·69	+1·32	5·85	76·9
14	154·71	0·89	6·59	2·71	7·12	67.6
15	166 86	0·86	7·22	3·99	8·25	61·0
16	179·01	0·83	7·53	5·10	9·09	55·8
17	191·17	−0·81	−7·45	+5·98	9·54	51·1
Dec. 28	325·19	−0·56	+7·34	−4·68	8·70	237·3
29	337·36	0·53	7·36	5·61	9·24	232·5
30	349·52	0·51	6·99	6·31	9·41	227·7
31	1·68	0·48	6·30	6·70	9·19	223·0
1898.						
Jan. 1	13·83	−0·45	+5·36	−6·78	8·63	218·1
2	25·98	0·43	4·22	6·56	7·80	212·6
3	38·12	0·40	2·96	6·05	6·74	206·0
4	50·26	0·37	1·63	5·29	5·54	197·1
5	62·39	0·34	+0·29	4·30	4·33	186·0
6	74·52	0·31	−1·03	3·13	3·29	161·7
7*	86·65	0·27	2·29	1·80	2·91	128·3
8	98·78	−0·24	3·44	−0·39	3·46	96·4
9	110·91	0·21	4·45	+1·06	4·58	76·6
10	123·04	0·18	5·28	2·48	5·83	64·8
11	135·18	0·15	5·95	3·80	7·06	57·3
12	147·32	0·12	6·36	4·96	8·06	51·9
13	159·46	0·08	−6·51	5·89	8·77	47·7
14	171·61	0·05	6·36	6·52	9·10	44·1
15	183·77	−0·03	5·90	+6·81	9·00	40·7
16	195·93	0·00	5·13	6·70	8·43	37 2
17	208·10	+0·03	4·05	6·19	7·39	33·0
18	220·28	0·06	2·71	5·28	5·94	27 0
19	232·46	0·08	−1·17	4·02	4·18	16·2
20	244·65	0·10	+0·46	2·47	2·51	349 5
21†	256·84	0·13	2·08	+0·76	2·21	290 0
22	269·03	0·15	3·56	−0·99	3·70	254·4

* During the small partial eclipse of the Moon on 1898 January 7. which, according to the *Nautical Almanac* for 1899, p. xiii. begins at $11^h\ 47^m{\cdot}4$ and ends at $13^h\ 22^m{\cdot}4$, the shadow does not seem to pass over any spot of well determined selenographical position.

† During the eclipse of January 21 the selenographical longitude Λ and latitude B of the point on the lunar surface in the centre of the shadow-cone

Greenwich Noon.	Selenographical Colong. of the Sun.	Lat.	Geocentric Libration. Sel. Long. of the Earth.	Lat.	Combined Amount.	Direction.
1898.	°	°	°	°	°	°
Jan. 23	281·23	0·18	4·81	2·65	5·49	241·1
24	293·42	0·20	5·74	4·10	7·05	234·3
25	305·61	0·23	6·29	5·28	8·20	229·9
26	317·79	+0·25	+ 6·44	−6·12	8·88	226·3
27	329·97	0·28	6·20	6·63	9·07	222·9
28	342·14	0·31	5·61	6·80	8·81	219·3
29	354·31	0·33	4·71	6·61	8·15	215·1
30	6·47	0·36	3·59	6·22	7·18	209·9
31	18·62	0·39	2·32	5·52	5·99	202·7
Feb. 1	30·77	+0·42	+0·97	−4·59	4·69	191·9
2	42·92	0·45	−0·39	3·46	3·49	173·6
3	55·06	0·48	1·67	2·18	2·75	142·5
4	67·20	0·51	2·83	−0·78	2·94	105·5
5	79·34	0·54	3·82	+0·67	3·87	80·1
6	91·47	0·57	4·58	2·11	5·04	65·3
7	103·60	+0·60	5·11	3·47	6·18	55·7
8	115·74	0·63	−5·38	4·68	7·13	48·9
9	127·88	0·66	5·41	5·67	7·83	43·5
10	140·02	0·69	5·20	6·36	8·21	39·1
11	152·17	0·72	4·78	6·72	8·23	35·3

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and the position angle P of the Moon's axis, reckoned as usually from the declination circle, will be

	h	°	°	
1898 Jan. 21	16 Gr. Λ =	+5·03	B = −0·12	P = 348·45
	18	4·02	0·12	·48
	20	3·00	0·13	·50
	22	+2·98	−0·13	348·53

The selenographical longitude λ and latitude β of the point on the rim of the Moon's disc in position angle p are found by

$$\sec s' \cos\beta \sin(\Lambda-\lambda) = \sin(p-\mathrm{P})$$
$$\sec s' \cos\beta \cos(\Lambda-\lambda) = -\cos(p-\mathrm{P}) \sin \mathrm{B} + \tan s' \cos B$$
$$\sec s' \sin\beta = \cos(p-\mathrm{P}) \cos \mathrm{B} + \tan s' \sin B,$$

where s' is the apparent semidiameter of the Moon's disc, or, if it is not worth while taking s' and B into account, if

$p-\mathrm{P}$ is between 0° and 180°, by $\lambda = \Lambda - 90°$ and $\beta = 90° - (p-\mathrm{P})$

and if

$p-\mathrm{P}$ is between 180° and 0°, by $\lambda = \Lambda + 90°$ and $\beta = 90° + p - \mathrm{P}$.

The position-angle of the axis of the Sun will be 351°·78 and the heliographical latitude of the centre of the disc—5°·39.

Greenwich Noon.	Selenographical Colong. of the Sun.	Lat.	Geocentric Libration. Sel. Long. of the Earth.	Lat.	Combined Amount.	Direction.
1898.	°	°	°	°	°	°
Feb. 12	164·32	0·74	4·16	6·69	7·87	31·8
13	176·48	0·76	3·38	6·28	7·12	28·2
14	188·65	+0·78	−2·46	+5·48	6·00	24·1
Feb. 24	310·59	+0·98	+5·06	−6·67	8·36	217·0
25	322·78	1·01	4·51	6·62	8·00	214·1
26	334·97	1·03	3·65	6·26	7·24	210·1
27	347·15	1·05	2·55	5·63	6·18	204·3
28	359·33	+1·07	+1·28	−4·76	4·93	195·0
Mar. 1	11·50	1·09	−0·08	3·70	3·70	178·7
2	23·66	1·11	1·44	2·47	2·86	149·8
3	35·82	1·14	2·72	−1·13	2·94	112·5
4	47·98	+1·16	−3·81	+0·28	3·82	85·7
5	60·13	1·18	4·66	1·71	4·96	69·9
6	72·28	1·20	5·21	3·08	6·05	59·4
7	84·43	1·22	5·44	4·32	6·94	51·4
8	96·57	1·24	5·33	5·36	7·55	44·7
9	108·72	+1·26	−4·91	+6·12	7·84	32·6
10	120·87	1·27	4·24	6·54	7·79	32·8
11	133·02	1·29	3·37	6·58	7·39	27·0
12	145·18	1·30	2·40	6·22	6·67	21·1
13	157·34	1·32	1·40	5·48	5·65	14·3
14	169·51	+1·33	−0·40	+4·40	4·42	5·2
Mar. 24	291·55	+1·42	+4·11	−6·53	7·71	212·1
25	303·76	1·43	3·47	6·23	7·13	209·0
26	315·97	1·44	2·57	5·66	6·21	204·3
27	328·18	1·45	1·44	4·83	5·04	196·5
28	340·38	1·46	+0·15	3·81	3·81	182·2
29	352·57	+1·47	−1·23	2·63	2·90	154·9
30	4·76	1·48	2·61	−1·33	2·93	117·1
31	16·95	1·49	3·89	+0·04	3·88	89·5
Apr. 1	29·13	1·50	4·97	1·42	5·17	74·0
2	41·30	1·50	5·77	2·77	6·40	64·3
3	53·47	+1·51	−6·22	+4·02	7·40	57·1
4	65·64	1·52	6·25	5·09	8·06	50·7
5	77·80	1·52	5·87	5·91	8·32	44·7
6	89·96	1·53	5·09	6·41	8·17	38·3
7	102·12	1·53	3·99	6·52	7·64	31·3

Greenwich Noon.	Selenographical Colong. of the Sun.	Lat.	Geocentric Libration. Sel. Long. of the Earth.	Lat.	Combined Amount.	Direction.
1898.	°	°	°	°	°	°
Apr. 8	114·28	+1·53	2·67	+6·22	6·77	23·1
9	126·45	1·53	−1·26	5·52	5·66	12·8
10	138·62	1·53	+0·12	4·46	4·47	358·4
11	150·80	1·53	1·39	3·12	3·42	336·1
12	162·98	1·53	2·47	+1·60	2·94	302·9
13	175·17	1·52	3·35	−0·00	3·35	269·9
14	187·37	1·52	4·03	1·58	4·33	248·5
15	199·58	+1·51	+4·51	−3·05	5·44	235·9

The selenographical positions of the Sun at the times assigned to the five published plates of the Lick Observatory Atlas of the Moon are the following ;—

Selenographical Colong. of the Sun.	Lat.	Assigned Pacific Standard Times.	h m s s	
°	°			
184·77	−0·93	1895 Oct. 10	16 49 10–17	Plate 1
172·64	−0·91	1895 Oct. 9	16 55 30–40	„ 2
159·56	−0·89	1895 Oct. 8	15 9 10–20	„ 3
172·62	−0·91	1895 Oct. 9	16 53 2–12	„ 4
61·07	−1·40	1896 Oct. 18	10 32 41–47	„ 5

The data for ascertaining the times when, for a number of spots represented on the photographs, the Sun reaches the zenith distances corresponding to those at the assigned times, are given in the following list, the data for plate 1 being already included in the list on page 90, under "Lick 6."

Sun's Colongitude. ⊙'s.	Spot.	Pl.	Sun's Colongitude. ⊙'s.	Spot.	Pl.
° 58·68−1·696 lat.	Horrebow	5	° 158·48−1·214 „	Cuvier	3
59·06−1·421 „	Bouguer	5	158·58−1·111 „	Tycho	3
59·34−1·220 „	Bianchini	5	158·59−1·093 „	Clairaut	3
59·47−1·136 „	Sharp	5	158·71−0·954 „	Maurolycus	3
59·70−0·969 „	C. Heraclides	5	158·93−0·706 „	Gemma Frisius	3
59·73−0·953 „	Harpalus	5	159·02−0·609 „	Aliacensis	3
59·87−0·849 „	Mairan	5	159·10−0·522 „	Apianus	3
60·16−0·639 „	Delisle	5	159·17−0·441 „	Sacrobosco	3
60·22−0·600 „	Wollaston	5	159·26−0·342 „	Geber	3
60·36−0·498 „	Euler	5	159·34−0·244 „	Abulfeda	3
60·38−0·480 „	Pytheas	5	159·38−0·210 „	Albategnius	3
60·45−0·442 „	Aristarchus	5	159·68−0·990 „	Stöfler	3

Sun's Colongitude. ⊙'s.		Spot.	Pl.
167°·73 − 5·50	lat.	Newton	2
170·08 − 2·837	,,	Moretus	2
170·58 − 2·287	,,	Blancanus	2
170·72 − 2·135	,,	Scheiner	2
171·11 − 1·698	,,	Clavius	2
171·11 − 0·586	,,	Aliacensis	2
171·23 − 1·570	,,	Longomontanus	2
171·37 − 1·394	,,	Lilius	2
171·52 − 1·235	,,	Maginus	2
171·75 − 0·989	,,	Tycho	2
171·77 − 0·957	,,	Stöfler	2
171·96 − 0·754	,,	Cichus	2

Sun's Colongitude. ⊙'s.		Spot.	Pl.
172°·05 − 0·650	,,	Hell	2
172·09 − 0·608	,,	Pitatus	2
172·17 − 0·494	,,	Purbach	4
172·26 − 0·401	,,	Thebit	4
172·36 − 0·292	,,	Alpetragius	4
172·41 − 0·235	,,	Alphonsus	4
172·44 − 0·201	,,	Albategnius	4
172·48 − 0·158	,,	Ptolemaeus	4
172·49 − 0·143	,,	Hipparchus	4
172·57 − 0·057	,,	Mösting A.	4
172·69 + 0·072	,,	Triesnecker A.	4
172·73 + 0·118	,,	Bode	4

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